Privately Optimal Securitization and Publicly Suboptimal Risk Sharing

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Abstract

Privately informed owners securitizing assets can signal positive information by retaining sufficient interest. Signaling provides social benefits, allowing uninformed investors to purchase securities without adverse selection. Instead of signaling, owners may prefer a pooling equilibrium in which they securitize the entire asset, relying on informed speculators to bring prices closer to fundamentals. Suboptimal risk sharing then results, since uninformed investors face adverse selection. We analyze privately optimal securitization and the choice between signaling and reliance on speculative markets. Prices are set competitively, with an endogenously informed speculator trading against rational uninformed investors. If a structuring exists providing the speculator sufficient gains, her effort is high, mispricing falls, and the entire asset is securitized in a pooling equilibrium. Risky debt and levered equity are an optimal bifurcation, with optimal face value trading off higher unit profits for the speculator against lower uninformed demand. Uninformed investors imperfectly insure, buying only debt, the only source of speculator profits. If risk-aversion is low or endowment shocks are small, uninformed demand is low, implying low speculator effort in pooling equilibria. High types then prefer separation, implying efficient risk sharing. The owner’s incentive to choose the separating equilibrium is weak when risk-aversion is high and/or endowment shocks are large, precisely when efficient risk sharing has high social value.

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Advocates of securitization commonly argue structured products improve risk sharing. For example, in recent testimony before the U.S. Senate, Goldman Sachs investment banker Fabrice Tourre stated: “To the average person, the utility of these products may not be obvious. But they permit sophisticated institutions to customize the exposures they wish to take in order to better manage the credit and market risks of their investment holdings.” The deeper intellectual argument in support of this benign view of financial innovation is the First Welfare Theorem which states that complete markets achieve Pareto optimal risk sharing (see e.g. Debreu (1959) and Arrow (1963)).

Policymakers have begun to question this positive view of securitization. For example, Adair Turner, Chairman of the U.K. Financial Services Authority, recently contended, “the argument that they created great allocative efficiency benefits via market completion was hugely overstated.” There is also a deeper intellectual argument buttressing this negative view of financial innovation, predicated on the fact that if markets remain incomplete, the introduction of a new security can actually make all agents worse off. For example, Hart (1975), Elul (1995) and Cass and Citanna (1998) show that in a symmetric information economy, opening a new security market can change relative prices of consumption goods, making all agents worse off. Dow (1998) shows that even if there is only a single consumption good, opening a new security market can make all agents worse off if there is asymmetric information across investors.

Although generic examples of welfare destroying financial innovation are suggestive, they do not allow one to take a completely informed view on the pros and cons of securitization, since they do not analyze which asset backed securities will actually be introduced in equilibrium. That is, they do not comprehend the methods and motives for securitization. The purpose of this paper is to develop a theory of privately optimal securitization when the issuer has private information, using a framework that permits welfare analysis. What forms of structuring are optimal for the issuer? And do private motives conflict with social welfare goals?

We consider the following setting. There are two periods and one consumption good. There is a single real asset generating verifiable cash flows in the second period, so the only publicly
traded securities are those backed by this asset. The securitization structure chosen by the asset’s owner influences risk sharing. The owner has an intrinsic motive to raise funds in the first period, being able to immediately convert each unit of numeraire received from investors into $\beta > 1$ units. However, he may also choose to hold a claim to a portion of the securitized asset’s cash flow. There is asymmetric information, with the owner knowing the true asset value, which is high or low, while other agents do not. Prices are set competitively by risk-neutral market-makers in response to market orders placed by rational risk-averse agents who are uninformed regarding the asset’s value and a risk-neutral speculator who can exert costly effort to increase the precision of the signal she receives regarding asset value.

The baseline model, which is of independent interest, considers the equilibrium securitization structure when/if the owner must sell the entire asset. For example, one may think of this setting as approximating a distressed bank. The baseline model is similar in spirit to that of Boot and Thakor (1993) with the most important exception being that the trading of the uninformed investors is rational. In addition to facilitating welfare analysis, which is impossible under pure noise-trading, endogeneity of uninformed trading leads to novel implications.

If the owner must sell the entire asset, no separating equilibrium exists, and the equilibrium structure is that preferred by the owner of a high quality asset. Consequently, under full securitization, the optimal structuring is that maximizing speculator effort, since this reduces expected underpricing for the high type. Two novel implications emerge from endogenous trading by uninformed investors. First, uninformed demand is confined to the least information-sensitive claim, e.g. debt, with all speculator gains coming from that market. Second, the optimal information sensitivity of the low information-sensitivity claim trades off two competing concerns. An increase in its information sensitivity increases the per-unit profit of the speculator, increasing her effort. However, this also induces an endogenous decline in uninformed demand for that claim, forcing the speculator to cut her own trade size. Under technical conditions, this tradeoff between per-unit profits and uninformed demand yields an interior optimal debt face value under full securitization.
In the general model, the owner can retain a claim to the asset’s future cash flow. This model is similar in spirit to that of Fulghieri and Lukin (2001) differing along three important dimensions. First, they consider exogenous noise-trading, precluding analysis of risk sharing and welfare. Second, in our extended model, any claim held by the original owner is optimally structured, whereas they assume the retained claim is ordinary equity. Finally, in our general model there exist fully-revealing separating equilibria in addition to pooling equilibria.

When the owner is allowed to retain an interest, the equilibrium set always includes the least-cost separating equilibrium (LCSE) as viewed by the high type. In the LCSE, the low type sells the entire asset in equity form while the high type sells only safe debt, retaining all risk on his balance sheet in the form of levered equity. This LCSE entails a deadweight loss since the high type underinvests. The prediction that low types finance with equity and high types finance with debt is standard in signaling models, as is the prediction that high types pass up positive NPV investments (see e.g. Myers and Majluf (1984)). However, we show that the LCSE has a socially attractive feature: it results in first-best risk sharing since full revelation of information eliminates adverse selection from securities markets. This fact has been overlooked in the signaling literature.

An important question is whether owners will implement separating equilibria when such equilibria are socially optimal. In the general model, there also exist pooling equilibria provided they weakly Pareto-dominate the LCSE from the perspective of both owner types. In the high type’s preferred pooling equilibrium (which we treat as the focal pooling equilibrium), the entire asset is sold to outside investors and bifurcated as described in the baseline model, maximizing speculator effort. The benefit of the pooling equilibrium relative to the LCSE is that expected investment is first-best. However, in this equilibrium uninformed investors face adverse selection, causing them to change their trading decisions, resulting in inefficient risk sharing.

The model provides a novel perspective on security design, one in which owners choose between signaling and reliance on speculative markets. With signaling, prices are equal to fundamentals, but a high type must retain sufficient exposure on his own balance sheet to make the signal credible, so
his fundraising is below first-best. With reliance on speculative markets, the high type can sell the entire asset, but he faces underpricing, with its severity hinging upon demand-side factors such as investor risk aversion and the size of endowment shocks.

This new perspective on security design highlights a fundamental conflict between private and public incentives in securitization, and in the degree of reliance on speculative markets. We show pooling equilibria exist if and only if the speculator can be incentivized to put in sufficient effort under the effort-maximizing bifurcation. Intuitively, the high type prefers pooling only if trading by the speculator will bring prices sufficiently close to fundamentals. *Intriguingly, factors that make efficient risk sharing more important from a social perspective increase the likelihood of private owners implementing pooling equilibria cum speculation and inefficient risk sharing.* To see this, note that the speculator hides behind the buy orders of uninformed investors, so their demand stimulates speculator effort. Thus, increases in the size of endowment shocks hitting uninformed investors or increases in their risk-aversion promote speculator effort. This makes the pooling equilibrium more attractive for the high type. Formally, we show that the LCSE payoffs cease to be the unique equilibrium payoffs when endowment shocks are large or risk-aversion is high.

We are by no means the first to analyze the choice between separating and pooling equilibria. However, standard analyses, e.g. Tirole (2005), assume investors revert to priors in the event of pooling. We show the pooling equilibrium is inherently more attractive to the high type than in canonical signaling models since speculative trading induces a favorable revision of beliefs away from priors. Further, in our model the choice between pooling and separating hinges upon demand-side factors outside the canonical signaling framework, such as the size of endowment shocks and investor risk-aversion. Finally, standard analyses are silent on risk-sharing since they assume universal risk-neutrality.

In addition to the papers cited above, our paper is most closely related to that of Gorton and Pennachi (1990). Their model begins with a very similar underlying tension. In their economy, there is initially insufficient riskless debt to meet the demand of uninformed investors who are averse to
trading at an informational disadvantage. They show uninformed investors have incentives to set up an intermediary that will pool endowments and bifurcate underlying cash flows into risky equity and riskless debt, with the latter being used by the uninformed investors as a store of value. Essentially, one can think of the uninformed investors as choosing financial structure for the intermediary in their model. In contrast, we determine the type of securities that will be issued by a privately-informed owner. On one hand, our results show there is actually greater scope for efficient risk sharing than is implied by their analysis, since we show that any separating equilibrium allows uninformed investors to trade securities (including equity) without fearing adverse selection. However, we also show that self-interested privately-informed asset owners have an incentive to deviate from the separating equilibrium, and sacrifice risk sharing, precisely when uninformed investors have the strongest insurance demands.

Our paper is also closely related to that of Allen and Gale (1988) who also evaluate optimal security design in a setting with endogenously incomplete markets, with the firm having a monopoly on issuing securities due to the need for asset-backing. The two models are predicated upon very different frictions. Allen and Gale assume symmetric information, but firms incur a cost when introducing a security. Hence, the tradeoffs in the models differ fundamentally.

DeMarzo and Duffie (1999) also analyze optimal security design from the perspective of an issuer who places intrinsic value on immediate liquidity. However, they consider a very different information structure. In their model, the issuer chooses the design of the security before observing the asset’s true value. After the structure is locked-in, the issuer observes the true asset value and decides how much to sell. Under technical conditions, e.g. monotonicity, debt is an optimal security since its low information-sensitivity results in low price impact per dollar raised. In contrast, we consider a setting where the issuer knows the asset’s value when choosing the amount to sell and the security design. Further, we allow for a speculator to acquire information about asset value, alleviating mispricing. Finally, the model of DeMarzo and Duffie is silent on risk sharing since they assume universal risk-neutrality.
Nachman and Noe (1994) analyze a setting, like ours, where the issuer is privately informed at the time the security is designed. In their setting, the scale of investment is fixed, and there is no possibility for separation or informed speculation. Under technical conditions, e.g. monotonicity, they show firms will pool at a debt contract, since debt minimizes the cross-subsidy from high to low types.

Hennessy (2008) analyzes optimal security design in a setting where the firm is originally owned by a continuum of uninformed investors who rationally trade the various claims on the firm after being hit with a preference shock. For such owners, it can be optimal to promote information acquisition to the extent that more informative prices help management make better investment decisions. However, it may be optimal to deter information acquisition, since this allows uninformed owners to sell without adverse selection. Dang, Gorton and Holmström (2010) analyze optimal security design in a more general setting than Hennessy (2008), but also rule out feedback effects on real decisions. Consequently, the optimal structure in their setting attempts to deter information acquisition to preserve efficient risk sharing. Both papers show debt is optimal for deterring information acquisition.

Price informativeness has been analyzed in other corporate finance settings. Holmström and Tirole (1993) present a model in which the equity float affects information acquisition, price informativeness, and the risk premia paid to managers. Aghion, Bolton and Tirole (2004) and Faure-Grimaud and Gromb (2004) show that price informativeness stemming from speculative monitoring can promote effort by insiders ex ante. In addition to the fact that all three models are predicated upon managerial moral hazard, the other critical difference with our approach is that uninformed trading is exogenous.

There is a voluminous literature on the nature of claims that will be introduced by securities exchanges seeking to maximize trading volume and/or profits earned on bid-ask spreads. These models depart from our working assumption, standard in the financial contracting and corporate finance literatures, that the issuer of the security is privately informed. Surveys of this line of
research are provided by Allen and Gale (1994) and Duffie and Rahi (1995). Within this literature, Marín and Rahi (2000) is related to our own model in showing a tension between private and public incentives in security design, but they derive their result in a radically different setting. In particular, they show that a volume maximizing futures exchange may leave markets incomplete even when market completeness is feasible. This occurs when the benefit of reduced adverse selection is swamped by the Hirshleifer (1971) effect.

The remainder of the paper is as follows. Section I describes the economic setting. Section II abstracts from the security design problem in order to describe the market-making process. Section III analyzes optimal structuring in the baseline model where the entire asset must be securitized. Section IV considers an extension of the baseline model in which the original owner may retain a claim on the underlying asset.

I. Economic Setting

This section describes preferences, endowments, and the market-making process. To provide a benchmark, we finish this section by describing equilibrium if information were symmetric.

A. Preferences and Endowments

There are two periods, 1 and 2, with a single nonstorable consumption good available in each period. This consumption good is the numeraire. The asset type ($\tau$) is either high ($H$) or low ($L$), with Owner being the only agent endowed with perfect knowledge of the type. The asset delivers $\tau$ units of the good in period 2 with perfect certainty, with $L \in (0, H)$. The uninformed prior probability of the asset being type $H$ is $q \in (0, 1)$.

Owner possesses the only tangible real asset in the economy. Owner has no endowment other than this real asset. The tangibility of the asset allows courts to verify its value in period 2. In contrast, the endowments of the various agents are not verifiable by courts. Consequently, other
agents cannot issue securities and cannot short-sell.\(^1\) Further, there can be no endowment-contingent contracts. Rather, courts can only enforce asset-backed payments contingent upon the observed value in \(\{L, H\}\). Since endowments are not verifiable by courts, Arrow securities are not possible and risk sharing may be inefficient. Allen and Gale (1988) also consider an incomplete markets setting in which the firm has the unique ability to issue securities that can help to improve risk sharing.

There is a measure one continuum of uninformed investors. These investors are analogous to defined benefit pension funds. Such pensions want to transfer funds across periods and do not particularly value high upside potential. Rather, they are very averse to being unable to meet their obligations to pensioners. Given such preferences, and lacking the resources to gather inside information in securities markets, they naturally seek relatively safe stores of value.

In the model, uninformed investors may have an insurance motive for purchasing securities delivering consumption in period 2. The uninformed investors are sufficiently wealthy in the aggregate to buy the entire asset since each has a first period endowment \(y_1 \geq H\). Uninformed investors face a common endowment shock, with their period 2 endowment \(y_2\) being either 0 or \(-\phi\). Just prior to securities market trading in period 1, uninformed investors privately observe a noisy signal regarding the size of their period 2 endowment. In particular, with probability one-half they find that they are “invulnerable” to a negative shock and will have endowment \(y_2 = 0\) with probability one. With probability one-half they are “vulnerable” to a negative shock. Conditional upon being vulnerable, they face a probability \(\eta \in (0, 1]\) of \(y_2 = -\phi\).

Uninformed investors are risk-neutral over first period consumption \(c_1\) and risk-averse over second period consumption \(c_2\). They are indexed by the intensity of their risk-aversion as captured by a preference parameter \(\theta\). The utility of uninformed investor-\(\theta\) takes the form:

\[
U(c_1, c_2; \theta) \equiv c_1 + \min(0, \theta c_2).
\]  

\(^1\)We could allow the marketmakers to supply a limited amount of risk-free assets without changing the qualitative results.
The preference parameters have compact support \( \Theta \equiv [0, \theta^{\text{max}}] \). Throughout, \( \theta^{\text{max}} \) is assumed to be sufficiently high such that there is always strictly positive uninformed demand demand for at least one security.\(^2\) The \( \theta \) parameters have density \( f \) with cumulative density \( F \). This distribution has no atoms, with \( f \) being strictly positive and continuously differentiable. As in Dow (1998), second period utility is piecewise linear, and has a concave kink at zero consumption. Since uninformed investors are averse to negative consumption in period 2, they have an intrinsic insurance motive for buying securities when vulnerable to negative endowment shocks.

There is a single speculator \( S \) who is risk-neutral and indifferent regarding the timing of consumption having utility equal to \( c_1 + c_2 \). In the first period, she is endowed \( y_1^{S} \geq H \) units of the numeraire, so she can afford to buy the entire asset. Her second period endowment is irrelevant and normalized at zero.

The speculator is unique in that she receives a noisy signal of asset type and can exert costly effort to increase signal precision. Letting \( s \in \{s_L, s_H\} \) denote the signal and \( \tau \) the true asset type, \( S \) chooses \( \sigma \equiv \Pr(s = s_\tau) \) from the feasible set \([1/2, 1]\). Her non-pecuniary effort cost function \( e \) is strictly positive, strictly increasing, strictly convex, twice continuously differentiable, and satisfies

\[
\begin{align*}
\lim_{\sigma \downarrow \frac{1}{2}} e(\sigma) &= 0 \\
\lim_{\sigma \downarrow \frac{1}{2}} e'(\sigma) &= 0 \\
\lim_{\sigma \uparrow 1} e'(\sigma) &= \infty.
\end{align*}
\]

If \( S \) puts in any effort, the signal becomes informative since

\[
\sigma > \frac{1}{2} \Rightarrow \Pr[\tau = H | s = s_H] = \frac{\Pr[\tau = H \cap s = s_H]}{\Pr[s = s_H]} = \frac{q \sigma}{q \sigma + (1 - q)(1 - \sigma)} > q. \quad (2)
\]

The final set of agents in the economy is a measure one continuum of market-makers. They are risk-neutral and indifferent regarding the timing of consumption having utility equal to \( c_1 + c_2 \). In the first period each market-maker is endowed with \( y_1^{MM} \geq H \) units of the numeraire, so they too

\(^2\)This avoids the need to continually check upper limits of integration when computing their demand.
can afford to buy the entire asset. Their second period endowment is irrelevant and normalized at zero.

**B. The Market-Making Game**

We characterize perfect Bayesian equilibria (PBE) of signaling games, requiring: all agents have a belief at each information set; strategies must be sequentially rational given beliefs; and beliefs are determined using Bayes’ rule and the equilibrium strategies for all information sets on the equilibrium path.

The market-making game is a signaling game played between the informed speculator and market-makers. The market-makers and uninformed investors enter the market-making game holding their prior belief that the asset has high quality with probability $\theta$. The market-making game starts with $S$ choosing $\sigma$ at personal cost $c(\sigma)$. Her choice of $\sigma$ is not observable, but is correctly inferred by other agents in equilibrium. Then $S$ privately observes the signal $s$. Next, the uninformed investors determine whether or not they are vulnerable to a negative endowment shock. Market orders are submitted and finally, market-makers set prices competitively. To do so, in the market-making game market-makers form beliefs regarding the signal received by $S$.

The market-making process is in the spirit of Kyle (1985) and Glosten and Milgrom (1985). The speculator and uninformed investors simultaneously submit non-negative market orders. Market-makers then set prices based upon observed aggregate demands in all markets. There is no market segmentation. At this information set, market-makers must have a belief about the signal $s$ for any aggregate demand configuration. Market-makers clear all markets, buying all securities not purchased by uninformed investors or the speculator.

Since Owner is the only agent capable of issuing claims delivering goods in period 2, market-makers cannot be called upon to take short positions. To this end, we impose the following technical assumption.

$$A1 : \phi \leq \frac{L}{2}.$$ 

The role of Assumption 1 is as follows. The aggregate demand of uninformed investors is weakly
increasing in $\phi$. Therefore, to avoid the possibility of aggregate demand exceeding supply for any security, the endowment shock must be sufficiently small. Sufficiency of Assumption 1 for ensuring no shorting by the market-makers is established below.

To summarize, in the baseline model the entire asset is securitized. The sequence of events in period 1 is as follows: Owner observes the asset type and chooses a structuring; the speculator exerts effort cost $e(\sigma)$ and then observes her signal; uninformed investors observe whether they are vulnerable to negative $y_L$; the speculator and uninformed investors simultaneously submit market orders; and finally market-makers observe aggregate demands in each market and set prices competitively. In period 2 endowments ($y_L$) are realized, the asset value is verified by the courts and the various claimants are paid.

C. Symmetric Information Benchmark

If information were symmetric Owner would sell the entire asset in the form of ordinary equity. The speculator would not put in any effort and would simply consume her first period endowment of $y_1^S$. The uninformed investors would not buy any claims to second period consumption if invulnerable. If vulnerable, a subset of uninformed investors would submit buy orders for the firm’s equity which would be correctly priced at $\tau$. Specifically, all uninformed investors with $\theta > \eta^{-1}$ would buy $\phi/\tau$ shares of equity ensuring $c_2 = 0$ in the event of a negative endowment shock and $c_2 = \phi$ if not. The remaining vulnerable uninformed investors would not buy any of the equity, implying $c_2 = -\phi$ in the event of a negative endowment shock and $c_2 = 0$ if not.

II. Market-Making with a Single Security

We set the stage for subsequent analysis by initially ignoring the security design problem altogether, focusing on how prices would be set by market-makers if the entire asset were sold as equity.

Many of the results derived in this section are relevant for cases where the owner bifurcates the asset into two securities. To handle bifurcation into two claims A and $B$, let $(A_L, A_H)$ and $(B_L, B_H)$
denote their respective period 2 payoffs as a function of the verified value in \{L, H\}. Security B is treated as the default in the case of only one security being issued. We have:

\[
\text{All Equity: } (B_L, B_H) = (L, H).
\]

Since she cannot short-sell, the optimal strategy for the speculator is to place a buy order if and only if she receives a positive signal. She attempts hiding her buy orders behind those of uninformed investors. The optimal size of her buy order is equal to the size of the aggregate buy order placed by uninformed investors when they are vulnerable to negative endowment shocks. This latter quantity is denoted \(X\).

Each uninformed investor conditions demand on his idiosyncratic preference parameter \(\theta\). An uninformed investor will not place a buy order if invulnerable to negative \(y_2\) since the marginal utility of any increase in \(c_2\) is then zero. An individual uninformed investor may place a buy order if vulnerable since there is an insurance motive to avoiding negative consumption. However, each uninformed investor is rational, weighing adverse selection costs against insurance motives when determining optimal demand.

Table 1 lists the possible aggregate demand configurations confronting market-makers. After observing aggregate demand, market-makers form beliefs regarding the signal received by the speculator based upon the observed aggregate demand \(D\), with:

\[
\begin{align*}
\Pr[s = s_H|D = 2X] &= 1 \\
\Pr[s = s_H|D = X] &= 1 - q - \sigma + 2q\sigma \\
\Pr[s = s_H|D = 0] &= 0.
\end{align*}
\]

Beliefs over \(s\) can be mapped to beliefs over the asset type, with

\[
\Pr[\tau = H|D] = \Pr[\tau = H|s = s_H] \Pr[s = s_H|D] + \Pr[\tau = H|s = s_L] \Pr[s = s_L|D] \quad (4)
\]
where

\[
\begin{align*}
\Pr[\tau = H|s = s_H] &= \frac{q\sigma}{1 - q - \sigma + 2q\sigma} \\
\Pr[\tau = H|s = s_L] &= \frac{q(1 - \sigma)}{q + \sigma - 2q\sigma}.
\end{align*}
\]

Substituting (5) into (4) one obtains:

\[
\begin{align*}
\Pr[\tau = H|D = 2X] &= \frac{q\sigma}{1 - q - \sigma + 2q\sigma} \\
\Pr[\tau = H|D = X] &= q \\
\Pr[\tau = H|D = 0] &= \frac{q(1 - \sigma)}{q + \sigma - 2q\sigma}.
\end{align*}
\]

Beliefs regarding \(\tau\) increase monotonically in aggregate demand with

\[
\Pr[\tau = H|D = 2X] > \Pr[\tau = H|D = X] > \Pr[\tau = H|D = 0].
\]

The market-makers set the price \((P)\) of equity as follows:

\[
P(D) = L + (H - L) \Pr[\tau = H|D] \quad \forall D \in \{0, X, 2X\}
\]

\[
\Rightarrow P(2X) > P(X) > P(0).
\]

To support the PBE conjectured in Table 1 it is sufficient to verify the speculator has no incentive to deviate regardless of the signal she receives. To that end, off the equilibrium path market-makers form adverse beliefs from the perspective of the speculator, setting prices based upon:

\[
\Pr[s = s_H|D] = 1 \quad D \notin \{0, X, 2X\}.
\]

It is readily verified that the speculator has no incentive to change her signal-contingent trading strategy when confronted with such beliefs. While such beliefs off the equilibrium path are sufficient to support the conjectured PBE of the market-making game, it is worthwhile to briefly discuss their plausibility. Note that any \(D \notin \{0, X, 2X\}\) must be due to the speculator placing a strictly positive order. The chosen specification of beliefs off the equilibrium path is predicated on the intuitive notion that market-makers should view any such (positive) order as being placed by \(S\) after having
observed \( s_H \). After all, if a negative signal is received, the speculator stands to incur a loss from buying securities unless the market-makers form the most favorable beliefs from her perspective, which would entail \( \Pr[s = s_H | D] = 0 \). Conversely, if a positive signal is received, the speculator stands to make a strictly positive trading gain provided \( \Pr[s = s_H | D] < 1 \).

A. Expected Revenue

The expected revenue of the owner, conditional upon \( \tau = H \), is given by:

\[
E[R|\tau = H] \equiv \bar{R}_H(\sigma) = L + (H - L) \left[ \frac{\sigma \Pr[\tau = H | D = 2X]}{2} + \frac{(1 - \sigma) \Pr[\tau = H | D = 0]}{2} + \frac{\Pr[\tau = H | D = X]}{2} \right].
\]  

Equation (10) can be rewritten as:

\[
\bar{R}_H(\sigma) = HZ(\sigma) + L[1 - Z(\sigma)] \\
Z(\sigma) = \frac{1}{2} \left[ \frac{q\sigma^2}{1 - q - \sigma + 2q\sigma} + \frac{q(1 - \sigma)^2}{q + \sigma - 2q\sigma} + q \right].
\]  

Anticipating, the variable \( Z \) plays an important role in the model. It measures the high type’s expectation of the market-makers’ updated belief. To take a limiting example, if it were possible to achieve \( Z = 1 \), then each type would receive the correct type-specific asset valuation.

From Bayes’ rule we may relate the expected revenue of the low type, denoted \( \bar{R}_L(\sigma) \), to that of the high type as follows

\[
E(\tau) = qH + (1 - q)L = q\bar{R}_H(\sigma) + (1 - q)\bar{R}_L(\sigma),
\]  

which implies

\[
\bar{R}_L(\sigma) = H\bar{z}(\sigma) + L[1 - \bar{z}(\sigma)] \\
\bar{z}(\sigma) = \left( \frac{q}{1 - q} \right) \left[ 1 - Z(\sigma) \right].
\]  

Intuition suggests \( \bar{R}_H \) and \( Z \) are increasing in the precision of the signal received by the speculator, as captured by \( \sigma \). For a high type, the more precise the signal, the more likely it is that
S observes $s_H$ and places a buy order. Since prices are increasing in aggregate demand, as shown in equation (8), expected revenue also increases with signal precision. To verify this conjecture, we compute

$$\frac{\partial R_{iy}}{\partial \sigma} = \frac{H - L}{2} \left[ \left( \Pr(\tau = H|2X) - \Pr(\tau = H|X) \right) + \left( \Pr(\tau = H|X) - \Pr(\tau = H|0) \right) \right] + \frac{1 - \sigma}{2} \left( \frac{\partial \Pr(\tau = H|0)}{\partial \sigma} \right).$$

The first line of equation (14) captures the direct benefit to a high type of an increase in signal precision. To see this, suppose first that the uninformed investors are indeed vulnerable, so their aggregate demand is $X$. If $S$ receives the correct signal, total aggregate demand observed by the market-makers then increases from $X$ to $2X$. Alternatively, if the uninformed investors are invulnerable, their aggregate demand is 0. If $S$ receives the correct signal, total aggregate demand observed by the market-makers then increases from 0 to $X$. The second line in equation (14) accounts for the effect of $\sigma$ on the belief revision process.

Lemma 1 confirms that the owner of a high value asset benefits from the speculator receiving a more precise signal. All but the most important proofs are presented in the appendix.

**Lemma 1** The expected revenue of the owner of a high value asset is increasing in the precision of the signal received by the speculator.

From Lemma 1 it follows that $Z$ is increasing in $\sigma$, with

$$Z(1/2) = q$$

$$Z(1) = \frac{1 + q}{2}.$$  

It is worth noting that if the speculator fails to put in effort, then $Z = q$ and the expected revenue of each type reverts to the unconditional expected revenue $E(R)$ as in a standard analysis of pooling equilibria.

**B. Incentive Compatible Information Acquisition**
Consider next the incentives of the speculator. From Table 1 it follows that her expected gross trading gain is

\[ G(\sigma, X) = X \cdot \left[ \frac{(q \sigma^2)}{2} [H - P(2X)] + (\frac{q \sigma^2}{2}) [H - P(X)] + (\frac{(1-q)(1-\sigma)}{2}) [L - P(2X)] + (\frac{(1-q)(1-\sigma)}{2}) [L - P(X)] \right] \]

\[ = \frac{q(1-q)(2\sigma - 1)(B_H - B_L)X}{2} \] (16)

It is readily verified that the speculator’s trading gain increases linearly in each of its arguments, and that the marginal benefit of signal precision is increasing in \( \sigma \), with

\[ G_1(\sigma, X) = q(1-q)(B_H - B_L)X > 0 \] (17)
\[ G_2(\sigma, X) = \frac{q(1-q)(2\sigma - 1)(B_H - B_L)}{2} > 0 \]
\[ G_{11}(\sigma, X) = G_{22}(\sigma, X) = 0 \]
\[ G_{12}(\sigma, X) = q(1-q)(B_H - B_L) > 0. \]

An incentive compatible signal precision, denoted \( \sigma_{ic} \) satisfies:

\[ e'(\sigma_{ic}) = q(1-q)(B_H - B_L)X. \] (18)

Define the inverse function of \( e' \) as follows

\[ \psi \equiv [e']^{-1}. \]

We may rewrite the incentive compatible signal precision as

\[ \sigma_{ic} = \psi[q(1-q)(B_H - B_L)X]. \] (19)

From the implicit function theorem and the convexity of the effort cost function \( e \) it follows that:

\[ \frac{\partial \sigma_{ic}}{\partial X} = \frac{q(1-q)(B_H - B_L)}{e''(\sigma_{ic})} \geq 0 \] (20)

\[ \frac{\partial \sigma_{ic}}{\partial (B_H - B_L)} = \frac{Xq(1-q)}{e''(\sigma_{ic})} > 0 \]

\[ \frac{\partial \sigma_{ic}}{\partial q} = \frac{X(B_H - B_L)(1-2q)}{e''(\sigma_{ic})}. \]
Since the incentive compatible signal precision plays a critical role, we summarize these findings in the following lemma.

**Lemma 2** The incentive compatible signal precision of the speculator is increasing in the aggregate demand of the uninformed investors when vulnerable \((X)\); increasing in the wedge between the value of claim \(B\) under high and low types \((B_H - B_L)\); increasing in \(q\) for \(q < 1/2\); and decreasing in \(q\) for \(q > 1/2\).

**C. Aggregate Uninformed Demand: All-Equity**

The next step is to determine aggregate uninformed demand \((X)\) for security \(B\) in response to their being vulnerable to a negative endowment shock. Before conducting this analysis it is worth recalling that if there were symmetric information regarding the asset’s value, uninformed investors with \(\theta > \eta^{-1}\) would fully insure against negative consumption in the second period. In particular, whenever vulnerable to negative endowment they would submit demand for \(\phi/\tau\) units of equity for an asset of type \(\tau \in \{L, H\}\). Such a demand would result in \(c_2 = 0\) if \(y_2 = -\phi\).

Letting \(x^*(\theta)\) denote the optimal \(\theta\)-contingent demand, aggregate uninformed demand demand is

\[
X \equiv \int_{0}^{\theta_{\text{max}}} x^*(\theta) f(\theta) \, d\theta. \tag{21}
\]

Each uninformed investor has measure zero and acts as a price-taker. If vulnerable, an individual uninformed investor expects the security to be overpriced since he knows a subset of uninformed investors will submit positive demands, pushing prices higher as market-makers revise upward their assessment of the probability of the asset being of high value. Despite facing adverse selection, an individual uninformed investor is willing to submit a buy order if \(\theta\) is sufficiently high.

In order to characterize uninformed demand, it is useful to compute the expected price of the asset conditional upon uninformed investors being vulnerable to a negative endowment shock. With this in mind, for the remainder of paper let \(\chi\) be an indicator function for uninformed investors
being vulnerable to a negative endowment shock.

\begin{equation}
E[P|\chi = 1] \equiv \underline{P}^-
\end{equation}

\begin{equation}
\underline{P}^- = [q\sigma + (1-q)(1-\sigma)]P(2X) + [q(1-\sigma) + \sigma(1-q)]P(X)
= qH + (1-q)L + q(1-q)(2\sigma - 1)(H - L).
\end{equation}

Equation (22) is consistent with the intuition that uninformed investors face adverse selection when submitting buy orders, since the asset is overpriced relative to its unconditional expected value. Additionally, the equation reveals there is no adverse selection if the speculator does not put in any effort ($\sigma = 1/2$). Finally, it can be seen that adverse selection is increasing in the precision of the speculator’s signal. This latter finding is at the heart of the trade-off in the model. Securitization structures that encourage information production by the speculator simultaneously worsen adverse selection as perceived by uninformed investors, biasing them away from first-best insurance against negative endowment shocks.

Recalling the functional form for uninformed investor utility in equation (1), consider the change in expected utility experienced by uninformed investors for various demand perturbations:

\begin{align}
x \in \left(0, \frac{\phi}{H}\right) &\implies \frac{\partial E(U|\chi = 1)}{\partial x} = \eta\theta[qH + (1-q)L] - \underline{P}^- \tag{23} \\
x \in \left(\frac{\phi}{H}, \frac{\phi}{L}\right) &\implies \frac{\partial E(U|\chi = 1)}{\partial x} = \eta\theta(1-q)L - \underline{P}^- \\
x > \frac{\phi}{L} &\implies \frac{\partial E(U|\chi = 1)}{\partial x} = -\underline{P}^-.
\end{align}

From the last line in (23) it is apparent that no uninformed investor ever buys more than $\phi/L$ units of equity since this results in $c_2 > 0$ with probability one and zero marginal consumption utility in the second period. In order to further characterize optimal uninformed demand, we define two cutoff values for the preference parameter:

\begin{align}
\theta_1 &\equiv \frac{\underline{P}^-}{\eta[qH + (1-q)L]}
\theta_2 &\equiv \frac{\underline{P}^-}{\eta(1-q)L}.
\end{align}

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From the demand perturbation equations (23) it follows that:

\[ \begin{align*}
\theta & \in [0, \theta_1] \Rightarrow x^*(\theta) = 0 \\
\theta & \in (\theta_1, \theta_2) \Rightarrow x^*(\theta) = \frac{\phi}{H} \\
\theta & \in [\theta_2, \theta_{\text{max}}] \Rightarrow x^*(\theta) = \frac{\phi}{L}.
\end{align*} \tag{25} \]

Adverse selection distorts trading decisions and the level of insurance against negative endowment shocks. Uninformed investors with relatively low risk-aversion (low \( \theta \)) go without any insurance against negative consumption. Such investors have \( c_2 = -\phi \) if there is a negative endowment shock. Information acquisition by the speculator induces more uninformed investors to forego insurance. To see this, note that \( \sigma > 1/2 \) implies \( \theta_1 > \eta^{-1} \), with the former representing the cutoff for the purchase of equity when there is asymmetric information and latter representing the cutoff for the purchase of equity under symmetric information. Intermediate preferences lead to partial underinsurance relative to first-best, with \( c_2 = 0 \) if \( \tau = H \) and \( c_2 = -\phi(H - L)/H \) if \( \tau = L \).

Lemma 3 summarizes.

**Lemma 3** If both types adopt an all-equity financial structure, aggregate uninformed demand when vulnerable to a negative endowment shock is

\[ \begin{align*}
X &= \phi \cdot \left[ \frac{F(\theta_2) - F(\theta_1)}{H} + \frac{1 - F(\theta_2)}{L} \right] \\
\theta_1 &= \eta^{-1} \left[ 1 + \frac{q(1 - q)(H - L)(2\sigma - 1)}{qH + (1 - q)L} \right] \\
\theta_2 &= \eta^{-1} \left[ 1 + \frac{q(1 - q)(H - L)(2\sigma - 1) + qH}{(1 - q)L} \right].
\end{align*} \tag{26} \]

Finally, we may pull together the incentive compatible signal precision from Lemma 2 and the uninformed demand from Lemma 3 to verify existence of equilibrium. We have the following proposition.
Proposition 1 In the market-making game, for any $B_H > B_L$, there exists a unique equilibrium pair $(\sigma^{eq}, X^{eq})$ satisfying

$$\sigma^{eq}(X^{eq}) = \sigma^{eq} \in \left(\frac{1}{2}, 1\right)$$

$$X(\sigma^{eq}) = X^{eq} \in \left(0, \frac{\phi}{B_L}\right).$$

III. Baseline Model: Optimal Structuring under Full Securitization

This section determines optimal security design in a baseline model predicated upon the assumption that Owner must sell the entire asset. In this case, there is no possibility of a separating equilibrium. This is because the owner of a low type asset would find it optimal to mimic whatever structuring would be chosen by the owner of a high type asset, since any other structure would fully reveal the negative private information, leaving him to collect the minimum expected revenue $L$. It follows that in the baseline model one may confine attention to the securitization structure that gives the high type the highest pooling payoff.

This baseline model is useful for three reasons. First, it approximates a number of real-world settings. For example, one may think of a distressed firm or financial institution that has an immediate need for cash. Alternatively, one may think of a conglomerate that has decided to spin off a division in order to focus on its core business. Finally, one may think of the baseline model as approximating a structured finance transaction where a bank has created a bankruptcy remote special purpose vehicle which will issue asset backed securities. The second reason for performing this baseline analysis is that it facilitates comparison with the findings of Boot and Thakor (1993) since they consider a setting in which all cash flow rights are sold to outside investors. Finally, solving the baseline model is a necessary precursor to Section IV which allows the original owner to endogenously retain some cash flow rights.
An important simplifying result is that attention can be confined to two publicly traded securities without loss of generality. This result is stated as Lemma 4.

**Lemma 4** Any outcome attainable with three or more publicly traded securities is attainable with two publicly traded securities.

Following Nachman and Noe (1994) and DeMarzo and Duffie (1999), attention is confined to securities with nonnegative payoffs that are weakly increasing in the asset value in period 2. Monotonicity is assumed for two reasons. First, monotone securities are commonplace. Second, if one of the securities, say security $A$, was decreasing, the owner of security $B$ could benefit at the expense of the owner of $A$ by making a clandestine contribution of additional funds to the asset pool. As argued by DeMarzo and Duffie (1999), only securities with monotone payoffs will be observed if such hidden contributions are feasible, and demanding monotonicity is then without loss of generality.

The major difference between the analysis of one and two securities is that uninformed investors insure themselves in the most efficient way possible. In particular, they buy the security that exposes them to the lowest degree of adverse selection per unit payoff. Therefore, we begin the analysis of multiple securities with a focus on uninformed demand.

**A. Uninformed Demand: Multiple Securities**

Uninformed investors have positive demands only if vulnerable to a negative endowment shock. By the same reasoning applied in Section II, uninformed investors are concerned about overpricing of securities in the event of a negative endowment shock since they correctly anticipate high aggregate demand.

Before proceeding, we first argue that in the baseline model where the original owner must sell off all cash flow rights, it is never optimal to issue a safe claim. To demonstrate this, suppose to the contrary Owner carves out a safe claim with $B_H = B_L \leq L$. Then all uninformed investors with $\theta > \eta^{-1}$ would use security $B$, and only $B$, to insure if vulnerable to a negative endowment shock,
setting their demand to \( x_B = \phi / B_L \). Such a financial structure would achieve first-best risk sharing. However, in the present setting, the goal of the owner (high type) is to encourage information production since this increases his expected revenue (Lemma 1). With safe debt available there will be no information production since there is no market in which the speculator can make trading gains. We state this result as Lemma 5.

**Lemma 5** If the owner were to issue a safe claim, uninformed demand would be confined to that claim, resulting in zero speculator effort \( (\sigma^{eq} = 1/2) \). Market-makers would then revert to their prior probability \( q \) of the asset being type \( H \). If the owner must securitize the entire asset, he will never issue a safe claim.

Lemma 5 highlights an inherent conflict between private and public incentives in the choice of securitization structures. In particular, the sale of a riskless claim would here achieve first-best risk sharing. However, if the owner must sell the entire asset, he necessarily finds it optimal to sacrifice risk sharing in order to encourage information acquisition.

In light of Lemma 5, the remainder of this section focuses on securitizations in which both claims are risky. To compute the optimal demand for each uninformed investor it is useful compute price expectations conditional upon vulnerability to a negative endowment shock. To this end, let:

\[
P_A^- = E[P_A | X = 1]
\]
\[
P_B^- = E[P_B | X = 1].
\]

Under this section’s working conjecture, to be verified, that all uninformed demand is concentrated in a single security, say security \( B \), there is zero aggregate demand for \( A \) since the speculator will also refrain from trading in that market given the lack of any cover provided by uninformed investors. Therefore, Table 1 continues to be the relevant table depicting aggregate demand (for security \( B \)). Since there is no market segmentation, the aggregate demand for security \( B \) is also used by the market-makers in setting prices for security \( A \). That is, the market-makers will set prices
as a function of aggregate demand for $B$ as follows:

\[
P_A(D) = A_L + (A_H - A_L) \Pr[\tau = H|D] \quad \forall \ D \in \{0, X, 2X\} \tag{27}
\]

\[
P_B(D) = B_L + (B_H - B_L) \Pr[\tau = H|D] \quad \forall \ D \in \{0, X, 2X\}.
\]

Recall, the only agent of positive measure is the speculator. Therefore, to support the perfect Bayesian equilibrium conjectured in Table 1, it is sufficient to verify that the speculator has no incentive to deviate. To that end, we assume the market-makers form adverse beliefs from the perspective of the informed trader, setting prices based upon $\Pr[s = s_H|D] = 1$ for any $D \notin \{0, X, 2X\}$ in the market for security $B$ and/or for any nonzero demand for security $A$. It is readily verified that the speculator then has no incentive to deviate from the conjectured signal-contingent trading rule.

Using Table 1 we arrive at the following expressions for the expected prices computed by uninformed investors when vulnerable to negative endowment shocks:

\[
\bar{P}_A = [q\sigma + (1 - q)(1 - \sigma)]P_A(2X) + [q(1 - \sigma) + \sigma(1 - q)]P_A(X)
\]

\[
= qA_H + (1 - q)A_L + q(1 - q)(2\sigma - 1)(A_H - A_L)
\]

and

\[
\bar{P}_B = [q\sigma + (1 - q)(1 - \sigma)]P_B(2X) + [q(1 - \sigma) + \sigma(1 - q)]P_B(X)
\]

\[
= qB_H + (1 - q)B_L + q(1 - q)(2\sigma - 1)(B_H - B_L).
\]

Equations (28) and (29) are consistent with the intuition that uninformed investors perceive overpricing for any security unless it is riskless. Further, the degree of perceived adverse selection is increasing in the precision of the speculator’s signal.

Assume without loss of generality that security $A$ is more informationally sensitive in the sense
of taking a larger percentage claim in the event that $\tau = H$:

$$
\frac{A_H}{H} \geq \frac{A_L}{L} \quad (30)
$$

$$
\Rightarrow \frac{P_A^-}{qA_H + (1 - q)A_L} \geq \frac{P_B^-}{qB_H + (1 - q)B_L}
$$

$$
\Rightarrow \frac{P_A^-}{(1 - q)A_L} \geq \frac{P_B^-}{(1 - q)B_L}.
$$

The two inequalities presented in (30) imply that security $A$ is viewed by uninformed investors as having higher adverse selection costs per unit of $c_2$ provided.

The intuition behind optimal uninformed demand is simple. For uninformed investors with $\theta$ sufficiently low, demand is zero for both securities, with adverse selection dominating insurance motives. For intermediate values of $\theta$, the agent partially insures, buying enough units of period 2 cash flow such that his consumption is zero if the actual asset type is $H$, which implies that consumption is negative if the actual asset type is $L$. Finally, if $\theta$ is sufficiently high, the uninformed investor completely insures in the sense of purchasing enough units of the security such that $c_2$ reaches zero even if the actual asset type is $L$, which implies $c_2 > 0$ if the asset type is $H$. Of course, for any given level of insurance, uninformed investors seek the least costly security combination.

Let $\Phi$ be an indicator for $y_2 = -\phi$. Second period consumption can then be expressed as a function of the actual asset type with

$$
c_2(x_A, x_B, \Phi, \tau) = x_A A_{\tau} + x_B B_{\tau} - \Phi \phi \quad \forall \quad \tau \in \{L, H\}.
$$

The optimal portfolio is determined using perturbation arguments. Attention is confined to portfolios satisfying $c_2(x_A, x_B, 1, L) \leq 0$. Otherwise, $c_2 > 0$ regardless of the actual asset type, despite the fact that the marginal utility of second period consumption is then equal to zero.

Consider an arbitrary portfolio such that $c_2(x_A, x_B, 1, H) < 0$ and evaluate a local perturbation. We have:

$$
\frac{\partial E[U|\chi = 1]}{\partial x_A} = \eta \theta [qA_H + (1 - q)A_L] - P_A^- \quad (31)
$$

$$
\frac{\partial E[U|\chi = 1]}{\partial x_B} = \eta \theta [qB_H + (1 - q)B_L] - P_B^-.
$$
If $\theta$ is sufficiently low, both perturbation gains listed in (31) are negative and optimal uninformed demand is zero. Specifically:

$$\theta \leq \frac{\mathcal{P}_B}{\eta[qB_H + (1-q)B_L]} \equiv \theta_1^B \iff (x_A^*, x_B^*) = (0, 0).$$  \hspace{1cm} (32)

Next, consider an arbitrary portfolio $(x_A, x_B)$ such that $c_2(x_A, x_B, 1, H) = \varepsilon$ where $\varepsilon$ is arbitrarily small. That is, we are considering points just above the kink that arises at portfolios such that $c_2(x_A, x_B, 1, H) = 0$. Performing a perturbation one finds:

$$\frac{\partial E[U|X = 1]}{\partial x_A} = \eta \theta (1-q)A_L - \mathcal{P}_A$$
$$\frac{\partial E[U|X = 1]}{\partial x_B} = \eta \theta (1-q)B_L - \mathcal{P}_B.$$ \hspace{1cm} (33)

If $\theta$ is sufficiently high, such a perturbation increases the maximand. Further, since the maximand is piece-wise linear, it would then be optimal to fully insure against negative consumption, achieving $c_2(x_A^*, x_B^*, 1, L) = 0$. Finally, from the inequality in (30), the minimal cost means of achieving this full insurance is to purchase only security $B$. Formally, we have:

$$\theta \geq \frac{\mathcal{P}_B}{\eta(1-q)B_L} \equiv \theta_2^B \iff (x_A^*, x_B^*) = \left(0, \frac{\phi}{B_L}\right).$$ \hspace{1cm} (34)

The final case to consider is $\theta \in (\theta_1^B, \theta_2^B)$. From the perturbation arguments given above, we know such uninformed investors partially insure, with $c_2(x_A^*, x_B^*, 1, H) = 0$. Now, consider the marginal utility ($MU$) per unit of period 1 numeraire allocated to the purchase of each security (on the relevant region where the uninformed investor is partially insuring). From the inequality in (30) we know:

$$MU_B = \eta[qB_H + (1-q)B_L] \geq \eta [qA_H + (1-q)A_L] = MU_A.$$ \hspace{1cm} (35)

It follows that security $B$ yields the highest marginal utility on the region of partial insurance, so that

$$\theta \in (\theta_1^B, \theta_2^B) \Rightarrow (x_A^*, x_B^*) = \left(0, \frac{\phi}{B_H}\right).$$ \hspace{1cm} (36)

This establishes Proposition 2.
Proposition 2 (Baseline Model) There is zero uninformed demand for the riskier claim $A$. When vulnerable to a negative endowment shock, aggregate uninformed demand for security $B$ is

$$X(\phi, B_L, B_H) = \phi \cdot \left[ \frac{F(\theta_2) - F(\theta_1)}{B_H} + \frac{1 - F(\theta_2)}{B_L} \right]$$

$$\theta_1^B = \eta^{-1} \left[ 1 + \frac{q(1-q)(B_H - B_L)(2\sigma - 1)}{qB_H + (1-q)B_L} \right]$$

$$\theta_2^B = \eta^{-1} \left[ 1 + \frac{q(1-q)(B_H - B_L)(2\sigma - 1) + qB_H}{(1-q)B_L} \right].$$

Proposition 2 can be contrasted with a result obtained by Boot and Thakor (1993). In their model, speculators make trading gains in the riskier levered equity claim. This results from their particular specification of noise trading. In our model, uninformed investors optimally insure themselves using only the least informationally sensitive claim. Consequently, in our model the speculator is unable to make trading gains in the market for the riskier claim. A similar effect is operative in the model of Gorton and Pennachi (1990), since they too predict that uninformed investors will hold only the safest claim, which is riskless debt in their model. However, in their model riskless debt is actually issued by the intermediary in equilibrium.

**B. Optimal Structuring**

In the baseline setting in which all cash flow rights must be sold, the sole objective of the owner of the high value asset is to maximize the incentive compatible level of signal precision $\sigma_{ic}$. To see this, recall from Lemma 1 that the expected revenue of such an owner is increasing in $\sigma$. Next, note that the incentive compatible signal precision is defined implicitly by equation (18). From convexity
of effort cost the function $e$ it follows that the optimal security design solves:

$$
\text{PROGRAM 1} \\
(B_L^*, B_H^*) \in \arg \max_{B_L, B_H} (B_H - B_l)X(\phi, B_L, B_H) \quad (38)
$$

subject to:

$$(\text{LIS}) \quad \frac{B_L}{L} \geq \frac{B_H}{H} \quad (39)$$

$$(\text{Monotonicity}) \quad B_H \geq B_L \quad (40)$$

$$(\text{Limited Liability}) \quad B_L \leq L.$$

Intuitively, the optimal financial structure under full securitization maximizes the product of the speculator’s per-unit profit and endogenous uninformed trading volume. This creates a natural trade-off given that uninformed demand decreases with informational sensitivity. The constraint labeled LIS ensures that security $B$ is, in fact, the low information-sensitivity security in which uninformed demand is concentrated. The three listed constraints ensure all other limited liability and monotonicity constraints are respected since they imply:

$$A_\tau(0, \tau) \in [0, \tau] \quad \forall \quad \tau \in \{L, H\}$$

$$B_L > 0$$

$$B_H \in (0, H]$$

$$A_H \geq \frac{HA_L}{L}.$$

Conveniently, Program 1 is independent of the choice of $B_L$ provided $B_L \in (0, L]$. This is because uninformed demand is homogeneous degree negative one in $(B_L, B_H)$. For example, if Owner were to cut both state contingent payoffs in half, each uninformed investor would simply double his demands. Thus, examining the objective function in Program 1, the optimal policy is unique up to a scalar, since

$$(B_H - B_L)X(\phi, B_L, B_H) = (\zeta B_H - \zeta B_L)X(\phi, \zeta B_L, \zeta B_H) \quad \forall \quad \zeta \in (0, 1]. \quad (41)$$
Given this finding, let

\[ B_H \equiv \kappa B_L. \]

In this case, the aggregate demand defined in Proposition 2 simplifies as follows (with slight abuse of notation):

\[ X = X(\phi, B_L, \kappa) = \frac{\phi}{B_L} \cdot \left[ 1 - \frac{F(\theta_1(\kappa))}{\kappa} - \frac{(\kappa - 1)F(\theta_2(\kappa))}{\kappa} \right] \quad (42) \]

\[ \theta_1(\kappa) \equiv \frac{1}{\eta} \left[ 1 + \frac{q(1-q)(2\sigma - 1)(\kappa - 1)}{1+q(\kappa - 1)} \right] \]

\[ \theta_2(\kappa) \equiv \frac{1}{\eta} \left[ 1 + \frac{q(1-q)(2\sigma - 1)(\kappa - 1) + q\kappa}{(1-q)} \right]. \]

Increases in \( \kappa \) reduce uninformed demand since both cutoffs are increasing in \( \kappa \) with:

\[ \theta'_1(\kappa) = \frac{q(1-q)(2\sigma - 1)}{\eta[1+q(\kappa - 1)]^2} > 0 \]

\[ \theta'_2(\kappa) = \eta^{-1} \left[ q(2\sigma - 1) + \frac{q}{(1-q)} \right] > 0. \]

Making the substitution \( B_H = \kappa B_L \) throughout Program 1 allows us to simplify the optimal structuring problem as follows.

**Lemma 6** Suppose total securitized cash flow is worth \( l \) if the asset is low quality and \( h \geq l \) if the asset is high quality. Total expected revenue received for the securitized claims, conditional upon the asset being of high quality, is maximized with any \( B^*_L \in (0, l] \) and \( B^*_H = \kappa^* B^*_L \) where \( \kappa^* \) solves

\[ \text{PROGRAM 2} \]

\[ \kappa^* \in \arg \max_{\kappa} M(\kappa) \equiv \phi(\kappa - 1) \left[ 1 - \frac{F(\theta_1(\kappa))}{\kappa} - \frac{(\kappa - 1)F(\theta_2(\kappa))}{\kappa} \right] \]

\[ \text{s.t.} \]

\[ \kappa \leq \frac{h}{l}. \]

It is worth noting that Program 2 is relevant for arbitrary levels of securitization, including cases where the owner retains some interest in the asset. Full securitization of the asset pertains to the special case where one sets \((l, h) = (L, H)\) in Program 2. The generality of Lemma 6 will prove useful in Section IV which considers partial securitization.
Lemma 7 establishes a sufficient condition under which the objective function in Program 2 is strictly concave.

**Lemma 7** If the cumulative distribution function $(F)$ for uninformed investors’ preference parameter $\theta$ is weakly convex, then the maximand in Program 2 $(M)$ is strictly concave.

The intuition behind Lemma 7 is as follows. If $F$ is convex then marginal increases in $\kappa$ result in ever larger reductions in aggregate uninformed demand. Further, the benefit to the speculator of the increase in per-unit profits stemming from an increase in $\kappa$ is spread over a progressively smaller trading base. Consequently, the maximand is strictly concave. For the remainder of the paper it is assumed that $F$ is convex.

$A2 : F$ is weakly convex.

The Lagrangian for Program 2 can be written as:

$$L(\kappa) = M(\kappa) + \lambda \left( \frac{h}{T} - \kappa \right).$$

(44)

In Program 2, the optimal policy is characterized by a unique pair $(\kappa^*, \lambda^*)$ satisfying the following first-order condition

$$M'(\kappa^*) = \lambda^*$$

(45)

and the complementary slackness conditions:

$$\left( \frac{h}{T} - \kappa^* \right) \lambda^* = 0$$

$$\lambda^* \geq 0.$$  

(46)

For the remainder of the analysis, we shall assume that $H/L$ is sufficiently high such that the LIS constraint does not bind if the asset is fully securitized. To this end, define $\kappa^{**}$ to be the unconstrained maximizer of the objective function $M$:

$$\kappa^{**} \equiv (M')^{-1}(0).$$
And we then adopt the technical assumption:

$$A3: \frac{H}{L} > \kappa^{**} \Rightarrow \kappa^*(L, H) = \kappa^{**}, \: \lambda^*(L, H) = 0.$$ 

One can understand the role of Assumption 3 as follows. Think of Owner as progressively raising \(\kappa\), bringing the low-information-sensitivity claim \(B\) closer and closer to a linear claim. Doing so raises the per-unit profit of the speculator, but also diminishes demand for \(B\), with Assumption 2 implying the demand cost rises with \(\kappa\). Assumption 3 is predicated on the notion that the demand cost dominates before \(B\) becomes linear. That is, Assumption 3 ensures that when Owner fully securitizes the underlying real asset, it is never optimal to package it as straight equity.

Differentiating the maximand yields:

$$M' (\kappa) = \phi \left[ 1 - \frac{F(\theta_1)}{\kappa} - \frac{(\kappa - 1)F(\theta_2)}{\kappa} \right] - \phi \left[ \frac{\kappa - 1}{\kappa} \left( \frac{F(\theta_2) - F(\theta_1)}{\kappa} + f(\theta_1)\theta_1'(\kappa) + (\kappa - 1)f(\theta_2)\theta_2'(\kappa) \right) \right].$$

(47)

The first term in (47) captures the gain from increasing informational sensitivity (via \(\kappa\)), as it increases the speculator's per-unit trading gain. The negative term captures the cost of increasing informational sensitivity in terms of reducing equilibrium uninformed demand, behind which the speculator hopes to hide her trading. Canceling terms one obtains:

$$M'(\kappa) = \phi \left[ 1 - F(\theta_2) \left( 1 - \frac{1}{\kappa^2} \right) - \frac{F(\theta_1)}{\kappa^2} - \frac{\kappa - 1}{\kappa} \left[ f(\theta_1)\theta_1'(\kappa) + (\kappa - 1)f(\theta_2)\theta_2'(\kappa) \right] \right].$$

(48)

We have then established the following proposition which characterizes optimal security design in the baseline model.

**Proposition 3** If the entire asset is securitized, the optimal structuring consists of a claim with low-information-sensitivity with \(B_L^* \in (0, L]\) and \(B_H^* = \kappa^{**}B_L^*\), where \(\kappa^{**} < H/L\) is the unique solution to

$$1 - \frac{F(\theta_1)}{\kappa^{**}} - \frac{(\kappa^{**} - 1)F(\theta_2)}{\kappa^{**}} = \left[ \frac{\kappa^{**} - 1}{\kappa^{**}} \right] \left[ \frac{F(\theta_2) - F(\theta_1)}{\kappa^{**}} + f(\theta_1)\theta_1'(\kappa^{**}) + (\kappa^{**} - 1)f(\theta_2)\theta_2'(\kappa^{**}) \right].$$

(49)
The second residual claim attracts zero aggregate uninformed demand. All informed trading gains are derived in the market for the low information-sensitivity claim.

The following corollary shows that under full securitization optimal structuring can be achieved by combining standard securities.

**Corollary (Baseline Model)** One optimal securitization structure consists of illiquid levered equity and liquid risky senior debt with face value $k^{**}L$. Another optimal securitization structure consists of liquid equity and an illiquid call option on the whole asset with strike price $k^{**}L$.

Since we have an analytical solution for uninformed demand, numerical illustrations are simple. Figure 1 depicts aggregate uninformed demand, conditional upon vulnerability, assuming that the distribution of $\theta$ parameters ($F$) is the uniform distribution with support $[0, 8]$, $q = 1/2$, and $\eta = 3/4$. As shown in the figure uninformed demand declines monotonically in informational sensitivity, as measured by $k$, since increases in $k$ induce marginal investors to either forego purchase of the security ($\theta_1$ increasing) or to purchase less units ($\theta_2$ increasing). Consistent with the fact that aggregate demand increases linearly in $\phi$, Figure 1 also shows that increases in the magnitude of endowment shocks induce outward shifts in the uninformed demand curves. For these same parameter values, Figure 2 plots the objective function $M$ for Program 2, which pins down the optimal informational sensitivity $k^{**}$. As shown in the figure, the optimal value of $k$ is actually independent of $\phi$, reflecting the fact that the maximand is linear in $\phi$. However, it is also apparent that higher values of $\phi$ result in correspondingly higher values of $M$, which implies higher incentive compatible signal precision.

At this point it is worth recalling the working assumption that the only party capable of issuing securities is Owner. That is, other agents cannot issue securities or short-sell. Now recall that the market-markets clear markets for all securities, buying one minus the combined aggregate demand of the uninformed investors and the speculator. But is it possible for aggregate demand to exceed supply? To address this question, notice that the maximum aggregate demand coming from the
uninformed investors and speculator is $2X$. We can write uninformed demand as:

$$X(\phi, B_L, \kappa) = \left( \frac{\phi}{B_L} \right) \left[ \frac{F(\theta_2) - F(\theta_1)}{\kappa} + 1 - F(\theta_2) \right].$$

Therefore, to avoid the possibility of market-makers being called upon to short-sell, $\phi$ must be sufficiently small in relation to $B_L$. The no-shorting constraint is clearly easiest to satisfy if Owner chooses $B_L = L$, where the choice of $B_L$ was otherwise arbitrary when we ignored the no-shorting constraint. From Assumption 1 it follows that the market-makers are never called upon to short-sell (even at suboptimal $\kappa$) since

$$\phi \leq L/2 \Rightarrow 2X(\phi, B_L^* = L, \kappa) < 1. \quad (50)$$

**IV. General Model: Optimal Degree and Design of Securitization**

The baseline model assumed the original asset owner must sell the entire asset. Such a setting is relevant when there is a forced asset sale due to antitrust enforcement, bankruptcy liquidation, or unbounded liquidity needs. This section considers an alternative setting in which the owner chooses both the degree and design for securitization of the original asset.

The assumptions for the remainder of the paper are as follows. Owner is risk-neutral and values consumption equally in both periods, having utility of the form $c_1 + c_2$. Further, Owner has access to a linear production technology allowing him to convert each unit of numeraire received from investors in period 1 into $\beta > 1$ units of numeraire in that same period. In contrast to the original real asset, the value of this short-term production technology is not verifiable by courts, so this stream of cash flow cannot be securitized.

We consider this particular setup for two reasons. First, it approximates a number of real-world settings. For example, one may think of a distressed bank as placing high, yet bounded, value on the immediate receipt of cash coming from securitization of an underlying asset. Second, this setup
allows us to retain our focus on the optimal securitization of a single real asset, here the originaleal asset with values in \( \{L, H\} \). This allows us to address how the option to retain some cash flow
rights affects the optimal securitization structure.

If there were no intrinsic benefit to receiving funds immediately \( (\beta = 1) \), the owner of the
high quality asset would not sell any claims on the real asset given asymmetric information. He
would then obtain his first-best payoff \( H \) by holding onto the entire asset. Conversely, if there were
symmetric information and if \( \beta \) were greater than one, then an owner of either asset type would sell
all cash flow rights (full securitization).

A. The Security Design Game

Maskin and Tirole (1992) show the equilibrium set of signaling games can be narrowed and
Pareto-improved (from the perspective of the privately informed party) by expanding the set of
feasible initial actions. Tirole (2005) describes an application of the formulation of Maskin and
Tirole (1992) to security issuance by a privately informed party. We adapt the game of Tirole
(2005) to our setting.

We characterize PBE again requiring: all agents have a belief at each information set; strategies
must be sequentially rational given beliefs; and beliefs are determined using Bayes’ rule and the
equilibrium strategies for all information sets on the equilibrium path.

The sequencing of events is as follows. The entire security design game actually consists of
two connected signaling games: an offer game and the market-making game. The latter game was
already described in Section I. The offer game is a signaling game played between Owner and all
outside investors. This game begins with Owner privately observing asset value. He then approaches
the market-makers (e.g. investment banks) and publicly proposes a menu of two securitization
structures, say \( \Sigma \in \{\Sigma^1, \Sigma^2\} \), that he would like the option to choose from subsequently. This
step resembles a shelf-registration in that Owner is locking in a pair of optional future financial
configurations. Each structure stipulates all payoffs for claimants as a function of the verified asset
value in period 2. The market-makers then agree to clear markets competitively for whatever
structure $\Sigma$ the owner subsequently chooses from his menu. All agents in the economy must have a belief regarding the asset type in response to any menu offer, including those off the equilibrium path. Beliefs at this stage are labeled offer beliefs. To support candidate PBE, menu offers off the equilibrium path are punished with outside investors inferring $\tau = L$ with probability one.

The sole difference between this formulation and the game described by Tirole (2005) is that in our model market-makers cannot agree to providing cross-subsidies. Rather, they simply agree to compete and clear markets for the securitization structure subsequently chosen by Owner. Using the terminology of Tirole (2005), competitive market-making implies that all investors in securities find them profitable type-by-type. This stands in contrast to a setting in which investors can pre-commit to subsequently buying some set of securities at a loss, a possibility allowed in the formulation of Tirole (2005).

In the next stage of the offer game, Owner selects a securitization structure $\Sigma$ from the menu he initially proposed, with the choice being incentive compatible. After observing the selection of Owner, all other agents revise beliefs using Bayes’ rule where possible. The beliefs formed at this stage are labeled selection beliefs. It is worth stressing that both types can offer the same menu, but they do not necessarily select the same securitization structure from that menu. Indeed, in a separating equilibrium of the offer game, the initial securitization proposal is such that the $\Sigma$ subsequently selected from the menu reveals the true asset type $\tau$. In any separating equilibrium securities are correctly priced and all agents trade in full knowledge of the true type. There is no incentive for the speculator to put in effort in a separating equilibrium of the offer game.

In a pooling equilibrium of the offer game, both owner types propose the same trivial menu with $\Sigma^1 = \Sigma^2$. In such cases, no information is revealed about the asset type after the selection stage of the offer game. If and only if a pooling equilibrium occurs in the offer game, play then passes to the market-making game described in Section I. Recall, all relevant players enter the market-making game holding their prior belief that $\Pr[\tau = H] = q$, as is appropriate when the offer game reveals no information regarding $\tau$. Then a signaling game ensues between the speculator and market-makers,
where market-makers use aggregate demand to form beliefs regarding the signal $s$ received by the speculator and set prices accordingly.

B. The Least-Cost Separating Equilibrium

In the general model, the owner can credibly signal positive private information by retaining sufficient rights. To this end, assume Owner designs a third security $C$ with value-contingent payoffs $(C_L, C_H)$. Owner holds security $C$ and sells the other two securities $A$ and $B$ to public investors in a competitive market. From Lemma 4 it follows that confining attention to no more than two publicly traded securities is without loss of generality.

We begin by evaluating the least-cost separating equilibrium (LCSE) from the perspective of the high type. Note that in the LCSE the speculator has no incentive to acquire information since the original owner’s private information is fully revealed by his financing choice.

The LCSE minimizes the low type’s incentive to mimic by giving him his first-best allocation in which he sells the entire asset in, say, equity form for $L$. The LCSE makes the high type as well off as possible subject to the constraint that the low type would not choose to mimic. In the LCSE, there is no need for the firm to sell more than one public security, call it security $B$. The LCSE is then the solution to:

$$\max_{(B_L, B_H, C_L, C_H)} \quad C_H + \beta B_H$$

s.t.

$$\text{No Mimic} : \quad \beta L \geq C_L + \beta B_H$$

$$\text{Limited Liability}$$

$$\text{Monotonicity}.$$ 

To determine the LCSE, we first ignore the monotonicity constraint and then verify it is slack. Clearly, in this relaxed program the optimal policy is to loosen the no-mimic constraint to the maximum extent by setting $C_L^* = 0$, implying $B_L^* = L$. Further, the no-mimic constraint must bind at the optimum, implying $B_H^* = L$ and $C_H^* = H - L$. Since the neglected monotonicity constraint
is satisfied we have established the following proposition.

**Proposition 4** In the least-cost separating equilibrium, a low type asset is sold in its entirety in all-equity form. The owner of a high type asset sells only a safe senior debt claim with face value $L$, retaining the residual levered equity claim. Uninformed investors then achieve first-best insurance against negative endowment shocks by purchasing correctly priced claims.

The intuition behind Proposition 4 is simple. In the LCSE, the low type would always mimic if the high type were to sell any risky claim since he would then benefit from security overvaluation. Therefore, the best the high type can do is to get the maximum liquidity possible subject to zero informational-sensitivity. Debt with face value $L$ achieves this objective.

In the LCSE, the high type experiences a loss relative to symmetric information equal to $(\beta - 1)(H - L)$. This deadweight loss reflects that fact that first-best entails him selling off the entire asset instead of just the claim to $L$. As in the model of Myers and Majluf (1984), in the LCSE asymmetric information results in the high type cutting back the scale of his investment to below first-best.

The socially attractive feature of the LCSE is that it achieves first-best risk sharing, regardless of the actual asset type. To see this, note that all marketed claims (the equity of the low type and the debt of the high type) are correctly priced since the equilibrium is fully-revealing. Therefore, the uninformed investors will insure themselves just as they did under perfect information. Specifically, all uninformed investors with $\theta > \eta^{-1}$ will purchase enough insurance such that they will achieve $c_2 = 0$ in the event of a negative endowment shock.

Proposition 4 shows the results in Gorton and Pennachi (1990) are overly restrictive in that their model relies upon safe debt to achieve perfect risk sharing. However, perfect risk sharing is achieved in any *separating* equilibrium, even if the separation is predicated upon the issuance of risky claims.

**C. The Equilibrium Set**

This subsection maps some of the results of Maskin and Tirole (1992) and Tirole (2005) to our setting, relying on somewhat different proofs due to differences in the economic settings considered.
The next lemma places a lower bound on what each type must receive in any equilibrium.

**Lemma 8** In any equilibrium of the security design game, each owner type must receive a payoff weakly greater than his least-cost separating payoff.

**Proof.** Suppose to the contrary that some type received less than his LCSE payoff. He could then profitably deviate by issuing safe debt with face value $L$ and retaining residual cash flow rights. □

The following lemma characterizes the equilibrium set.

**Lemma 9** The equilibrium set of the security design game always includes the least-cost separating equilibrium. It also includes pooling equilibria with a single contract on the offered menu provided that contract weakly Pareto dominates the least-cost separating equilibrium (from the perspective of both owner types).

**Proof.** Consider first supporting the LCSE. If beliefs were set to $\Pr[\tau = H] = 0$ in response to any deviating menu, then no such deviation is profitable. Suppose next there is a pooling contract weakly Pareto dominating the LCSE. If beliefs were set to $\Pr[\tau = H] = 0$ in response to any deviating menu, the deviator would get weakly less than his LCSE payoff and the deviation is not profitable. □

**C. The Pooling Equilibrium**

Consider next the nature of pooling equilibrium—an equilibrium in which both types offer a trivial menu such that $\Sigma^1 = \Sigma^2$. Confining attention to pooling equilibria, the best such equilibrium from the perspective of the high type maximizes

$$C_H + E[P_A + P_B|\tau = H].$$

Any pair $(C_L, C_H)$ held by the original owner leaves a total residual stream $(l, h)$ of payments that will be packaged and sold to outside investors:

$$(C_L, C_H) \Rightarrow (l, h) \equiv (L - C_L, H - C_H).$$
We characterize the optimal nature and scope of securitization using a two step procedure. First, Lemma 6 can be used to characterize the optimal structuring for the sale of residual cash flows after netting out the retained claim. Then \((C_L, C_H)\) are optimized in light of their effect on the value attainable in this residual structuring problem.

Before proceeding with the formal solution, it is useful to sketch the intuition. For the owner of a high quality asset, the benefit of increasing \(C_H\) is that he marginally reduces his exposure to underpricing. However, this retention of cash flow rights from the long-term tangible real asset reduces the amount he can invest in the profitable short-term project. Further, an increase in \(C_H\) reduces \(h/l\). If \(h/l \leq \kappa^*\), where \(\kappa^*\) is defined in Proposition 3, the LIS constraint in Program 2 is binding and the incentive compatible signal precision falls below that attainable under full securitization.

Let \(M^*(l, h)\) denote the maximum value obtained in Program 2 given that the total value of publicly traded claims on the real asset is in \(\{l, h\}\):

\[
M^*(l, h) \equiv M[\kappa^*(l, h)].
\]  

From (18) and the definition of \(M^*\) it follows that the maximized incentive compatible signal precision is:

\[
\sigma^*(l, h) \equiv \psi[q(1 - q)M^*(l, h)].
\]  

From the Envelope Theorem we know:

\[
M_1^*(l, h) = \frac{\partial L}{\partial l} = \frac{-h\lambda^*(l, h)}{l^2} \Rightarrow \sigma_1^*(l, h) = \frac{-q(1 - q)h\lambda^*(l, h)}{l^2\psi'(\sigma^*)} \leq 0
\]

\[
M_2^*(l, h) = \frac{\partial L}{\partial h} = \frac{\lambda^*(l, h)}{l} \Rightarrow \sigma_2^*(l, h) = \frac{q(1 - q)\lambda^*(l, h)}{l\psi'(\sigma^*)} \geq 0.
\]

The inequalities in (54) convey an important trade-off. Specifically, when the LIS constraint is binding, increases in \(l\) reduce the value obtained in Program 2 and with it the incentive compatible signal precision \(\sigma^*\). Conversely, increases in \(h\) loosen the LIS constraint, potentially leading to higher \(\sigma^*\). Thus, consistent with the intuition provided above, a high value of \(C_H\) imposes a cost in
terms of the power of incentives that can be provided to the speculator. Lower incentives then lead
to more severe mispricing of the public claims.

With this in mind, we turn to the solution of the following restated program characterizing the
preferred pooling equilibrium for the high type.

\[
\begin{align*}
\text{PROGRAM 3} \\
\max_{l,h} & \quad \mu(l,h) \equiv H - h + \beta[l + (h - l)Z(\sigma^*(l, h))] \\
\text{s.t.} & \\
\text{Incentive Compatability} & : \quad \sigma^*(l, h) = \psi[q(1 - q)M^*(l, h)] \\
\text{Monotonicity} & : \quad h \geq l \\
\text{Limited Liability} & : \quad h \in [0, H] \text{ and } l \in [0, L].
\end{align*}
\]

Program 3 is not necessarily concave. Therefore, instead of relying on first-order conditions, we
pin down the optimal policy via perturbation and dominance arguments.

Casual intuition suggests the optimal pooling contract for the owner of a high quality asset
entails \(C_L^* = 0\) and leaving public investors with a total payoff in the low state equal to \(l^* = L\).
After all, the owner of a high quality asset has no desire to retain any cash flow rights should the
observed asset value be equal to \(L\), since he knows this is a zero probability event. He also knows
that the concomitant increase in \(l\) tends to boost his revenues, since investors are paying for the
rights to \(l\). However, there is a countervailing cost to such a policy, since increases in \(l\) tighten the
\(LIS\) constraint, potentially reducing the incentive compatible level of signal precision. Formally, we
can see these competing effects at work since

\[
\mu_1(l, h) = \beta[1 - Z(\sigma^*(l, h))] + Z'(\sigma^*(l, h))\sigma_1^*(l, h)]. \quad (55)
\]

The following lemma shows that the liquidity effect dominates in that the low payoff is always fully
securitized in the high type’s preferred pooling equilibrium.
Lemma 10 The optimal pooling contract for the high type entails a zero payoff to the owner if asset value is low ($C_L^r = 0$ and $l^* = L$).

Lemma 10 allows us to rewrite Program 3 as a one dimensional optimization:

$$
\text{PROGRAM 3'}}
\max_h \mu(L, h) \equiv H - h + \beta[L + (h - L)Z(\sigma^*(L, h))]
\text{s.t.}
\quad \text{Incentive Compatability} : \quad \sigma^*(L, h) = \psi[q(1 - q)M^*(L, h)]
\quad \text{LL&Mono} : \quad h \in [L, H].
$$

It is readily verified that if the owner of the high quality asset opts to pool at a structuring in which only safe debt is securitized he gets the same payoff as what he attains under the LCSE contractual structure, with

$$
\mu(L, L) = H - L + \beta L. \quad (56)
$$

Further, if $\beta Z[\sigma^*(L, H)] \leq 1$ the optimal pooling contract for the high type is to pool at a structuring in which only safe debt is securitized. To see this, note that

$$
\beta Z[\sigma^*(L, H)] \leq 1 \Rightarrow \mu(L, h) \leq \mu(L, L) \quad \forall \ h \in (L, H]. \quad (57)
$$

This leads directly to Proposition 5.

Proposition 5 If $\beta Z[\sigma^*(L, H)] \leq 1$ the payoffs and outcomes under the least-cost separating contract are the unique payoffs and outcomes.

The intuition for Proposition 5 is straightforward. If the speculator cannot be incentivized to produce sufficiently precise signals, even when her incentives are maximized under full asset securitization, then the costs of underpriced securities exceed the value of immediate liquidity and the owner of a high quality asset avoids issuing any risky security. Rather, he gets the maximal liquidity possible using safe debt.
Recall, the objective in Program 3 is to find the pooling contract preferred by the high type. Apparently, if \( \beta Z[\sigma^*(L, H)] \leq 1 \) it is impossible to find a pooling contract that makes him better off. And since we ignored the welfare of the low type in that program, it follows that there is no Pareto-improving contract across the owner types when \( \beta Z \leq 1 \). Also, the actual outcome for all agents under the pooling contract described in Proposition 5 is identical to that under the LCSE.

Consider next the optimal pooling contract for the high type when the speculator can be incentivized to produce more precise signals, in the sense that \( \beta Z[\sigma^*(L, H)] \geq 1 \). To analyze this case, note first that the maximand in Program 3 is linear once \( h \) is sufficiently high. In particular,

\[
\forall \ h \in (L\kappa^{**}, H), \quad \mu_2(L, h) = \beta Z(\sigma^*(L, H)) - 1
\]

which is a strictly positive constant in the posited scenario. It follows that:

\[
\beta Z(\sigma^*(L, H)) > 1 \Rightarrow \mu(L, H) > \mu(L, h) \quad \forall \ h \in [L\kappa^{**}, H).
\]

And further, using the fact that the maximand is linear for high values of \( h \) we know:

\[
\beta Z[\sigma^*(L, H)] > 1
\]

\[
\Downarrow
\]

\[
\mu(L, L\kappa^{**}) = \mu(L, H) - [H - L\kappa^{**}][\beta Z(\sigma^*(L, H)) - 1] = H - L + \beta L + (L\kappa^{**} - L)[\beta Z(\sigma^*(L, H)) - 1] \geq \mu(L, h) \quad \forall h \in [L, L\kappa^{**}).
\]

Proposition 6 follows immediately from the inequalities in (59) and (60).

**Proposition 6** If \( \beta Z[\sigma^*(L, H)] > 1 \), the preferred pooling contract for the high type entails full securitization of the asset, with optimal bifurcation of the asset following Proposition 3. Under this structuring, information is produced by the speculator and uninformed investors are imperfectly insured. Both owner types are strictly better off under this pooling contract than under the least-cost separating equilibrium, so the latter is not a unique equilibrium of the security design game.

Propositions 5 and 6 taken together offer a complete characterization of the equilibrium set.

The former predicts that when the information maximizing structuring described in Proposition 3
is insufficient for inducing high speculator effort (σ), the high type finds it impossible to improve upon the LCSE. In such cases, the high type will underinvest, securitizing only safe debt and keeping all risk on his own books. In contrast, when the structuring described in Proposition 3 does induce sufficiently high speculator effort, both types may respond by pooling at that same structuring, with the entire asset sold off to outside investors.

D. Private versus Public Incentives in Securitization

We are particularly interested in determining whether privately optimal securitization structures align with the social optimum. We here consider a social planner placing equal weight on all agents in the economy. The main argument would carry over for any weighting with strictly positive weights on all agents.

To set a benchmark, consider first social welfare under symmetric information. Here, the owner would sell the entire asset in the first period, regardless of type, converting each unit of funds raised into β > 1 units of consumption. The speculator and market-makers would consume their endowments. If vulnerable to a negative endowment shock, all uninformed investors with θ > η⁻¹ would spend φ units of first period numeraire in order to insure against negative consumption. The remaining uninformed investors would go without insurance and incur utility losses associated with negative consumption. The implied ex ante social welfare is:

\[ W^{FB} = \beta[qH + (1 - q)L] + y^S_1 + y^{MM}_1 + y_1 - \frac{\phi}{2} [1 - F(\eta^{-1})] - \frac{n\phi}{2} \int_0^{\eta^{-1}} \theta f(\theta) d\theta. \]  

Ex ante, the planner computes the following welfare loss in the LCSE relative to first-best:

\[ DWL^{SEP} = q(\beta - 1)(H - L). \]  

The only deadweight loss in the LCSE is the loss in NPV resulting from the high type operating the new short-term project below optimal scale. From a risk sharing perspective the LCSE is attractive, since full revelation of information results in first-best risk sharing.

Consider next the pooling equilibrium in which the asset is fully securitized under the corresponding optimal structuring described in Proposition 3. Note first that the equalities in equation
(12) imply that the expected level of investment in the pooling equilibrium is equal to the socially optimal level. However, this equilibrium also entails costly effort on the part of the speculator and results in inefficient risk sharing. The calculation of social welfare in the pooling equilibrium is a bit more involved. As a first step it can be computed as:

\[
W^{\text{POOL}} = \beta[qH + (1 - q)L] + y_1^S + y_1^{MM} + y_1 + [G - e]
\]

\[
= \frac{\eta \phi}{2} \left[ \int_{\theta_1}^{\theta_2} \left( (1 - \kappa^{-1}) \frac{e}{\theta f(\theta)} + (1 - q) \right) \theta f(\theta) d\theta \right]
\]

\[
= \frac{1}{2} [1 - F(\theta_2)] \left( \frac{\phi}{B_L P_B} \right) - \frac{1}{2} [F(\theta_2) - F(\theta_1)] \left( \frac{\phi}{B_H P_B} \right).
\]

The first line in equation (63) measures the value of aggregate investment, plus the first period endowments plus net trading gains to the speculator. The second line measures the costs incurred by the uninformed investors when they have negative \( c_2 \). The third line measures the expected units of \( c_1 \) that are spent by the uninformed investors when they purchase their optimal portfolios. This equation simplifies as follows:

\[
W^{\text{POOL}} = \beta[qH + (1 - q)L] + y_1^S + y_1^{MM} + y_1 - e - \frac{\phi}{2} [1 - F(\theta_1)] - \frac{\eta \phi}{2} \int_{0}^{\theta_1} \theta f(\theta) d\theta
\]

\[
= \frac{1}{2} (1 - q) \left( 1 - \kappa^{-1} \right) \left( \int_{\theta_1}^{\theta_2} (\theta - 1) f(\theta) d\theta - \frac{\phi}{2} q [\kappa - 1] [1 - F(\theta_2)] \right)
\]

\[
+ \frac{1}{2} (1 - q) \left( 1 - \kappa^{-1} \right) [F(\theta_2) - F(\theta_1)].
\]

Subtracting (64) from (61) one obtains the following expression for the deadweight loss in the pooling equilibrium:

\[
DWL^{\text{POOL}} = e[\sigma^*(\phi)] + \frac{\phi}{2} \left[ \int_{\theta_1}^{\theta_2} (\eta \theta - 1) f(\theta) d\theta + (1 - q) \left( 1 - \kappa^{-1} \right) \int_{\theta_1}^{\theta_2} (\eta \theta - 1) f(\theta) d\theta + q(\kappa - 1) [1 - F(\theta_2)] \right]
\]

Equation (65) has the following intuition. The first term reflects the fact that speculator effort is socially costly. The term in large square brackets reflects the fact that the existence of asymmetric information in the pooling equilibrium leads to distortions in the portfolios of the uninformed
investors relative to first-best. The first term in the large brackets captures the fact that a socially inefficient number of uninformed investors to forego insurance altogether. The second term in the large brackets reflects the fact that adverse selection induces a socially inefficient number of uninformed investors to only partially insure against negative consumption. And the final term represents the social cost associated with overinsurance ($c_2 > 0$) by extremely risk averse uninformed investors.

From equation (65) it is readily verified that the deadweight loss associated with the pooling equilibrium is increasing in the size of endowment shocks as follows:

$$
\frac{\partial DWL_{POOL}}{\partial \phi} = e'(\sigma^*) \frac{\partial \sigma^*}{\partial \phi} + \frac{1}{2} \left[ \int_{\theta_1}^{\theta_2} (\eta \theta - 1) f(\theta) d\theta + (1 - q) (1 - \kappa^{-1}) \int_{\theta_1}^{\theta_2} (\eta \theta - 1) f(\theta) d\theta + q(\kappa - 1)[1 - F(\theta_2)] \right]
$$

Note that the deadweight loss in the separating equilibrium is independent of $\phi$, but increasing in $\beta$. Conversely, the deadweight loss under pooling is independent of $\beta$, but increasing in $\phi$. It follows that by equating the deadweight losses across the two types of equilibria we may pin down a critical value of $\beta$, call it $\beta_{\text{public}}$, at which the social planner would be just indifferent between the two equilibria. Specifically:

$$
\beta_{\text{public}}(\phi) = \frac{e[\sigma^*(\phi)] + \phi}{q(H - L)}
$$

It is readily verified that $\beta_{\text{public}}$ is increasing in $\phi$. Intuitively, an increase in $\phi$ raises the risk sharing cost associated with the pooling equilibrium, so that the only way to maintain social planner indifference is to have a compensating increase in $\beta$, which raises the deadweight cost of the underinvestment associated with the separating equilibrium.

Similarly, we may pin down a critical value of $\beta$, call it $\beta_{\text{private}}$ at which the high type would be just indifferent between the two equilibria. From Proposition 6 we know that the indifference region is determined by:

$$
\beta_{\text{private}}(\phi) = [Z(\sigma(\phi))]^{-1} \Rightarrow \frac{d\beta_{\text{private}}}{d\phi} = -[Z(\sigma(\phi))]^{-2} \left[ \frac{\partial \sigma}{\partial \phi} \right] < 0.
$$
In contrast to the social planner, the high type is more attracted to the pooling equilibrium for higher values of $\phi$ since large endowment shocks stimulate uninformed demand and speculator effort, resulting in less underpricing in the pooling equilibrium. Hence, to maintain indifference for the high type, a compensating decrease in $\beta$ is required in response to a marginal increase in $\phi$.

Figure 3 pulls the entire analysis together in a convenient way. The solid upward sloping line depicts $\beta_{public}$. The social planner prefers the separating equilibrium at points to the right of $\beta_{public}$ reflecting the fact that high values of $\phi$ are associated with large losses due to inefficient risk sharing. The dotted downward sloping line depicts $\beta_{private}$. The high type prefers the pooling equilibrium at points to the right of $\beta_{private}$, reflecting the fact that large endowment shocks stimulate speculator effort and mitigate the extent of underpricing he faces in the pooling equilibrium.

Private and public preferences are conflicting on Regions 1 and 4. On Region 1 the planner prefers pooling, since the low $\phi$ values imply low risk sharing distortions under pooling. However, the high type here prefers the separating equilibrium, recognizing that he will face severe underpricing if he pools with the low type, given that low $\phi$ values induce low speculator effort. Conversely, on Region 4 the planner prefers the pooling equilibrium, with the high $\phi$ values raising the risk sharing costs of the pooling equilibrium. However, on this same region the high type prefers pooling, since he recognizes that high $\phi$ values stimulate speculator effort and mitigate the extent of underpricing he will face.

Similarly, an increase in risk-aversion via a first-order stochastic dominant shift in $\theta$ would also increase the welfare loss associated with the pooling equilibrium, while simultaneously making that equilibrium more attractive to the owner of a high quality asset. Taken together, these results indicate that the private sector will often prefer the pooling equilibrium, with high volumes of securitized assets and inefficient risk sharing, precisely when this risk sharing has the highest social value.
Conclusions

This paper evaluates privately optimal securitization structures when the original asset owner has an intrinsic motive for raising funds immediately, but is concerned about mispricing given that he is privately informed regarding asset value. Securities markets are endogenously incomplete, with the securitization structure influencing risk sharing. Prices are set by competitive market-makers, with an endogenously informed speculator trading against uninformed investors placing rational orders. If the speculator can be sufficiently incentivized, the extent of underpricing is reduced and the entire asset is securitized in a pooling equilibrium. Here, all speculator gains are derived in the market for the less information-sensitive claim, with the optimal structuring maximizing the product of the speculator’s per-unit gain and uninformed trading volume. Uninformed investors imperfectly insure in pooling equilibria, as adverse selection distorts their trading decisions.

There also exists a separating equilibrium in which a low type sells the entire asset in equity form while a high type only sells safe debt, holding levered equity on his own books. In this separating equilibrium, the type is fully revealed so there is no motive for costly information acquisition by the speculator and perfect risk sharing is achieved since uninformed investors do not fear adverse selection. However, there is a social cost to the separating equilibrium, since the high type operates below efficient scale.

The model highlights the following fundamental conflict between private and social incentives in choosing securitization structures: Private incentives to implement the pooling equilibrium are strongest precisely when the gains to efficient risk sharing are highest. Specifically, increases in the size of endowment shocks and/or risk-aversion encourage owners to rely upon speculative activity, rather than signaling, since higher uninformed trading volume subsidizes information acquisition by speculators, which reduces the extent of mispricing. Thus, the private sector will engage in socially excessive securitization and distort risk sharing when society most highly values the risk sharing benefits that advocates commonly attribute to such structures.

With this being said, it would be incorrect to interpret our model as implying that securitization
and financial innovation are bad things. In our model, the bifurcation of cash flows into streams with varying risk characteristics serve vital social purposes. For example, bifurcation is also necessary to support separating equilibria with efficient risk sharing. Further, it would be incorrect to interpret this model as supporting proposals forcing banks to keep more skin in the game since compulsory compliance prevents socially valuable information revelation via signaling.

A correct interpretation of this analysis is that private owners can be biased against financial structures that improve risk sharing. This is because they fail to internalize the externality they impose on uninformed investors when they issue informationally-sensitive securities. In fact, our analysis shows that even when privately-informed owners can insulate uninformed investors from adverse selection, they have an incentive to issue securities with nonzero informational-sensitivity in order to promote information production by speculators. Further, our analysis shows that when endowment shocks are large or risk aversion is high, private owners will not engage in enough signaling via retention of risk on their own balance sheets (i.e. will engage in excessive securitization), because they fail to internalize the social benefits signaling provides in terms of improved risk sharing. We leave the issue of optimal policy responses as a promising avenue for future research.
Appendix: Proofs

Lemma 1

Substituting beliefs from equation (6) into the expected revenue (10), we obtain:

\[
R_H(\sigma) = L + \left( \frac{q(H - L)}{2} \right) \left[ \frac{\sigma^2}{1 - q - \sigma + 2\sigma q} + q + \frac{(1 - \sigma)^2}{q + \sigma - 2\sigma q} \right].
\] (67)

We need only verify the square bracketed term is increasing. Let

\[
a(\sigma) \equiv q + \sigma - 2q
\]

\[
\Omega(\sigma) \equiv 1 + \frac{\sigma^2}{(1 - a)} + \frac{(1 - \sigma)^2}{a}.
\]

We need only verify \(\Omega\) is increasing. Differentiating we obtain:

\[
\Omega'(\sigma) = \frac{2(1 - a)\sigma + (1 - 2q)\sigma^2}{(1 - a)^2} - \frac{2a(1 - \sigma) + (1 - 2q)(1 - \sigma)^2}{a^2}
\]

\[
= \frac{[2(1 - a) + (1 - 2q)\sigma] \sigma a^2 - (1 - a)^2(1 - \sigma) [2a + (1 - 2q)(1 - \sigma)]}{(1 - a)^2 a^2}
\] (68)

This is strictly positive if and only if.

\[
[2(1 - a) + \sigma(1 - 2q)] \sigma a^2 > (1 - a)^2(1 - \sigma) [2a + (1 - 2q) - \sigma(1 - 2q)]
\]

\[
\downarrow
\]

\[
[(1 - a) + (1 - q)] \sigma a^2 > (1 - a)^2(1 - \sigma) [a + (1 - q)]
\]

\[
\downarrow
\]

\[
(1 - q)\sigma a^2 > (1 - a) [(1 - \sigma)(1 - a) + (1 - \sigma)(1 - q)(1 - a) - \sigma a^2]
\]

\[
\downarrow
\]

\[
(1 - q)\sigma a^2 > (1 - a) [a(1 - a) - a\sigma + (1 - \sigma)(1 - q)(1 - a)]
\]

\[
\downarrow
\]

\[
[(1 - q)a + 1 - a] \sigma a > (1 - a)^2 [a + (1 - \sigma)(1 - q)]
\]

\[
\downarrow
\]

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\[
[1 - qa] \sigma a > (1 - a)^2(1 - q \sigma)
\]
\[\downarrow\]
\[
\sigma a - qa^2 > (1 - a)^2 - q \sigma(1 - a)^2
\]
\[\downarrow\]
\[
q \sigma [(1 - a)^2 - a^2] + \sigma a > (1 - a)^2
\]
\[\downarrow\]
\[
q \sigma + \sigma a(1 - 2q) > (1 - a)^2
\]
\[\downarrow\]
\[
a^2 + q(\sigma - a) > (1 - a)^2
\]
\[\downarrow\]
\[
q^2(2\sigma - 1) + 2[\sigma - q(2\sigma - 1)] > 1
\]
\[\downarrow\]
\[
(q - 1)^2(2\sigma - 1) - (2\sigma - 1) + 2\sigma > 1
\]
\[\downarrow\]
\[
(q - 1)^2(2\sigma - 1) > 0.
\]

Proposition 1

Consider a graph with \( X \) on the vertical axis and \( \sigma \) on the horizontal axis. Plotting aggregate uninformed demand, we know \( X(1/2) > 0 \) and that \( X \) is strictly decreasing in \( \sigma \) on \([1/2, 1]\). Plotting the incentive compatible signal precision, we know \( \sigma_{ic} \) is strictly increasing in \( X \) with \( \sigma_{ic}^{-1}(1/2) = 0 \) and the limit as \( \sigma_1 \) converges to one of \( \sigma_{ic}^{-1}(\sigma_1) = \infty \). Thus, the two curves intersect once, and only once, implying a unique equilibrium exists.

Lemma 4

Suppose Owner sells \( N \geq 3 \) securities. Rank these securities in descending order in terms of the
ratio of their payoff if value is low relative to their payoff if value is high. Section III establishes that
hedge trading will be concentrated in security 1, and security 1 will be the only source of informed
trading gains. Aggregate demand of the uninformed investors and informed trader will then be zero
in securities 2 to \( N \). Therefore, one may roll up these securities into a single security having no effect
on \( \sigma \) or expected revenues.■

**Lemma 7**

Differentiating the maximand one obtains

\[
\frac{M'(\kappa)}{\phi} = 1 - F(\theta) + \kappa^{-2}[F(\theta_2) - F(\theta_1)] - (1 - \kappa^{-1})[f(\theta_1) \theta'_1 + f(\theta_2) \theta'_2 (\kappa - 1)].
\]

And

\[
\frac{M''(\kappa)}{\phi} = -2\kappa^{-2} f(\theta_1) \theta'_1 - 2\kappa^{-3}[F(\theta_2) - F(\theta_1)]
- (1 - \kappa^{-1})[f'(\theta_1)(\theta'_1)^2 + f'(\theta_2)(\theta'_2)^2]
- (1 - \kappa^{-1})[2(1 + \kappa^{-1}) f(\theta_2) \theta'_2 - f(\theta_1) |\theta'_1|].
\]

Since \( F \) is convex, a sufficient condition for \( M'' < 0 \) is \( |\theta'_1| \leq \theta'_2 \), which always holds.■

**Lemma 10**

This lemma is proved in a series of steps. First, we claim

\[
h^* = H \Rightarrow l^* = L.
\]

And

\[
l^* < L \Rightarrow h^* < H.
\]

To demonstrate this, note

\[
h^* = H \Rightarrow \forall l \in (0, L), \ \lambda^*(l, h^*) = 0 \Rightarrow \mu_1(l, h^*) > 0.
\]

Next we claim

\[
\lambda^*(l^*, h^*) = 0 \Rightarrow l^* = L.
\]

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To demonstrate this suppose to the contrary that \((l_0, h_0)\) are optimal with \(\lambda^*(l_0, h_0) = 0\) but \(l_0 < L\). Then consider increasing \(l\) by \(\varepsilon\) arbitrarily small, noting that such an increase meets all constraints including monotonicity since \(\lambda^*(l_0, h_0) = 0\) implies \(h_0 > l_0\). The gain is \(\varepsilon \beta (1 - Z) > 0\), contradicting the initial conjecture.

Next we claim

\[
\lambda^*(l^*, h^*) > 0 \Rightarrow l^* = L.
\]

To demonstrate this claim, suppose to the contrary that \((l_0, h_0)\) are optimal with \(\lambda^*(l_0, h_0) > 0\) but \(l_0 < L\). Then let \(\kappa_0 \equiv h_0/l_0\) and consider all pairs \((l, \kappa_0 l)\). By construction, all such pairs keep \(Z\) fixed at

\[
Z[\sigma^*(l_0, h_0)] \equiv Z_0.
\]

Then consider

\[
\frac{d}{dl} \mu[l, \kappa_0 l] = \beta (1 - Z_0) + \kappa_0 [\beta Z_0 - 1] \quad \forall \quad l \in (0, L).
\]

Note that the value of this derivative is constant by construction. We next claim the derivative must be weakly positive. For if it is not, the optimal policy is to decrease both \(l\) and \(h\) to zero leaving the owner to collect \(\mu = H\) which is strictly dominated by \(l = h = L\). Finally, since the derivative is weakly positive \(l^* = L\).
References


### Table 1: Aggregate Demand Outcomes

<table>
<thead>
<tr>
<th>Type</th>
<th>Signal</th>
<th>Vulnerable</th>
<th>Informed Demand</th>
<th>Hedge Demand</th>
<th>Aggregate Demand</th>
<th>Probability</th>
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<td>$Y$</td>
<td>$X$</td>
<td>$X$</td>
<td>$2X$</td>
<td>$rac{q_\sigma}{2}$</td>
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<td>$\frac{(1-q)\sigma}{2}$</td>
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