Demographics and The Behaviour of Interest Rates

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February 2012

Abstract

This paper takes a new step in the direction of linking financial variables to the age structure of population. A demographic variable, namely the ratio of middle aged to young population, is used to model the common persistent component of the term structure of interest rates. The age composition of the population determines the equilibrium nominal rate in the monetary policy rule and therefore the persistent component in 1-period yields. Fluctuations in demographics are then reflected in the whole term structure via the expected policy rates components in the yields at maturities longer than 1-period. Our specification does not impose strong priors on the relative importance of demographics for the determination of the equilibrium real rate and the inflation target as we use it to model the sum of these two variables. The empirical evidence shows that there is a strong effect of demographics on the persistent component of inflation. The presence of demographics in short-term rates allows more precise forecast of future policy rates, especially at very long-horizon, and helps modelling the entire term structure. Term structure macro-finance models with demographics clearly dominate traditional term-structure macro-finance models.

J.E.L. CLASSIFICATION NUMBERS: C33, C53, E43, E44

Keywords: demographics, central bank reaction function, term-structure models, forecasting

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1 Introduction

The behavior of interest rates is consistent with the decomposition of spot rates in the sum of two processes, (i) a very persistent long term expected value and (ii) a mean-reverting component. This paper investigates the hypothesis that the persistent long-term component of interest rates depends on the age structure of the population. The monetary policy maker sets the policy rate by reacting to cyclical swings reflected in the transitory (expected) variation of output from its potential level and of (expected) inflation from its target but slow moving changes in the economy that take place at generational frequency are also watched. We argue that the Fed sets the target by taking into account the saving behaviour and the preferences for inflation of the population. When young adults, who are net borrowers, and the retired, who are dissavers, dominate the economy savings decline and interest rates rise[1], moreover increases in the population of net savers dampen the preference for inflation. Linking the target policy rates to demographics makes Taylor-type rule of monetary policy capable of generating the observed persistence in interest rates. Yields at different maturities depend on the sum of short rate expectations and the risk premium. While it is not plausible to consider the risk premium as a non-mean reverting component, the presence of a persistent component related to demographics can be rationalized in terms of smooth adjustments in short-rate expectations that take decades to unfold. In particular, we consider MY, defined as the ratio of middle-aged (40-49) to young (20-29) population in the US as the relevant demographic variable to determine the persistent component of interest rates.

To our knowledge, the potential relation between demographics and the target policy rate in a reaction function has never been explored in the literature. This is the main contribution of our paper that is relevant for two reasons.

First, the persistence of policy rates cannot be modeled by the mainstream approach to central bank reaction functions that relate monetary policy exclusively to cyclical variables. Second, putting term structure model at work to relate the policy rate to all other yields requires very long term projections for policy rates. In a monthly model, 120 step ahead predictions of the 1-month rate are needed to generate the 10-year yield. In a specification where the persistent component of the policy rates is captured by demographics and macroeconomic factors are used to model the cyclical fluctuations, long-term projections are feasible as a VAR could used to project the stationary component, while the permanent component is projected without an econometric model by exploiting the exogeneity of the demographic variable and its high predictability even for very long-horizons (the Bureau of Census currently publishes on its website projections for the age structure of the population with a forecasting horizon up to fifty years ahead).

[1] The idea, is certainly not a new one as it can be traced in the work of Wicksell(1936) and Keynes(1936), but it has received little attention in the recent literature.

The trend-cycle decomposition of rates has been also recently investigated by Fama(2006) and Cieslak and Povala(2011), who argue that the predictive power of the forward rates for yield at different maturities could be related to the capability of appropriate transformations of the forward rates to capture deviations of yields from their permanent component. These authors propose time-series based backward looking empirical measures of the persistent component; in particular Fama considers a 5-year backward looking moving average of past interest rates and Cieslak and Povala consider a ten-year discounted backward-looking moving average of annual core CPI inflation.

Figure 1 illustrates the existence of a persistent component in interest rates and the possibility of relating it to demographic trends. The Figure reports the yield to maturity of 1-Year US Treasury bond, the persistent components as identified by Fama and Cieslak and Povala, and MY.

Insert Figure 1 about here

The Figure shows that MY not only strongly co-moves with the alternative estimates of the persistent component, but it is also capable of matching exactly the observed peak in yields at the beginning of the eighties, while, by its nature, the backward looking moving averages lag behind.

The very persistent component of yields is common to the entire term structure of interest rates: Figure 2 illustrates this point by reporting for US post-war data, the relationship between nominal interest rates at different maturities and MY_t.

Insert Figure 2 about here

The visual evidence reported in Figures 1-2 motivates the formal investigation of the relative properties of the different observable counterparts for the unobservable persistent component of the term structure.

After placing our contribution in the literature we shall implement the formal investigation in four stages.

First, a simple model of the components of the yield curve is proposed to illustrate the potential of the temporary-permanent decomposition to explain fluctuations of the term structure.

Second, we shall document the relative performance of MY and the backward looking measures proposed in the literature to model the persistent component of interest rates.

Third, we adopt a more formal representation of our simple framework, by estimating full term structure models. We consider first a weak Expectations Theory in which risk premia are not explicitly modelled and then an affine term structure model with time-varying risk premium. Having established the importance of demographics for term structure models, a section is then devoted to exploit the peculiar feature of a term structure with exogenous demographic factors and produce long-term forecast for the yield curve up to 2050.

Fourth, we investigate if the importance of demographics in determining an equilibrium rate in the central bank reaction function depends on the capability of demographics to capture the persistent component of inflation.
Finally, after a section devoted to assessing robustness of our empirical findings, the last section concludes and discusses agenda for further research.

2 Related Literature

This paper models interest rates as the sum of a persistent long-term expected value and a mean-reverting cyclical component. The short-end of the term structure (the one-period yield) is determined by monetary policy. In setting the policy rate, the Fed reacts to cyclical swings reflected in transitory variations of the deviation of output from potential output and inflation from its (implicit) target. This response determines the mean reverting cyclical component of one-period rates. However, monetary policy not only stabilizes cyclical fluctuations, but is also anchored to a long-term equilibrium rate. We argue that this long-term equilibrium rate is determined by the Fed by watching slow-moving changes in the economy that take place at a generational frequency, i.e. those spanning several decades, and we relate this to the age structure of population, MY_t as it determines savings behavior and the preferences for inflation of the population. All yields to maturity higher than one-period can be expressed as the sum of average expected future short rates and the term premium. Therefore, they can also be decomposed in the sum of a persistent component, reflecting the expected age structure of the population over the life of the bond, and a mean reverting cyclical component, reflecting expected transitory variations in economic cycle and inflation, and the (stationary) term premium.
Our framework brings together four different strands of the literature: i) the one analyzing the implication of the presence of a persistent component for spot rates predictability, ii) the one linking demographic fluctuations with asset prices. iii) the empirical literature modelling central bank reaction functions using the rule originally proposed by Taylor(1993) and iv) the term structure models with observable macro factors and latent variables.
The literature on spot rates predictability has emerged from a view in which forecastability is determined by the slowly mean-reverting nature of the relevant process. Recently, it moved to a consensus that modelling a persistent component is a necessary requirement for a good predictive performance.

Traditionally, yield curve modelling in finance is governed by parsimony principle; all the relevant information to price bonds at any given point in time is summarized by a small number of stationary factors (Litterman and Scheinkman, 1991). Both macro and finance term structure models agree on the role of at least two factors, i.e. the level and the slope, that capture a slow mean-reverting component and a more rapid (business-cycle-length) mean-reverting component. Besides providing a structural interpretation, this literature also documents the role of forward interest rates in forecasting future spot rates for longer horizons (e.g. Fama and Bliss, 1979). Particularly linear combinations of forward rates are successful in predicting excess returns (Cochrane and Piazzesi, 2005). Early literature attributes this predictability to the mean reversion of the spot rate toward a constant expected value. This view has been recently challenged; the predictability of the spot rate captured by forward rates is either attributed to a slowly moving, yet still stationary, mean (Balduzzi, Das, and Foresi, 1998) or to the reversion of spot rates towards a time-varying very persistent long-term expected value (Fama, 2006, Cieslak and Povala, 2011). These authors considers backward looking measures of the persistent component, in particular Fama considers a 5-year backward looking moving average of past interest rates and Cieslak and Povala consider a ten-year discounted backward-looking moving average of annual CPI inflation. We propose to use a forward looking measure based on a variable, MY, for which reliable forecasts are available for all the relevant horizons. The use of this variable allows us to explicitly model the change of regime in the spot rate that, following Hamilton (1988), have been modeled via regime-switching specifications for the spot rate (for example, Gray (1996), Ang and Bekaert (2002)).

Our choice of the variable determining the persistent component in short term rate is funded in the literature linking demographic fluctuations with asset prices and in the empirical approach to central bank reaction functions based on Taylor's rule.

Taylor's rule model policy rates as depending on a long term equilibrium rate and cyclical fluctuations in (expected) output and inflation.

The long-term equilibrium is traditionally modeled as a constant, we relate it to the age structure of population. The long term equilibrium rate is the sum of two components: the equilibrium real rate and equilibrium inflation, which is the (implicit) inflation target of the central bank. Both components can be related to demographics.

Geanakoplos, Magill and Quinzii (2004, henceforth, GMQ) consider an overlapping generation model in which the demographic structure mimics the pattern of live births in the U.S., that have featured alternating twenty-year periods of boom and busts. They conjecture that the life-cycle portfolio behavior (Bakshi and Chen, 1994), which suggests that agents should borrow when young, invest for retirement when middle-aged, and live off their investment once they are retired, plays an important role in determining equilibrium asset prices. Consumption smoothing by the agents, given the assumed demographic structure requires that when the MY_t ratio is small (large), there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged) and for the market to clear, equilibrium prices of financial assets should adjust, i.e. decrease (increase), so that saving (consumption) is encouraged for the middle-aged (young). The model predicts that the price of all financial assets should be positively related to MY_t and it therefore also predicts the negative correlations between yields and MY_t . Note that we use the results of the GMQ model to rationalize the target for policy rate at generational frequency, in this framework there is no particular reason why the ratio of middle-aged to young population should be directly linked to aggregate risk aversion. Following this intuition, we take a different approach from the available literature that studied the relationship between real bond prices and demographics through the impact on risk aversion(Brooks 1998; Bergantino, 1998; Davis and Li, 2003). In a separate strand of literature, Lindh and Malberg (2000) investigate the hypothesis that inflation pressures covary with the age distribution. The results of the estimation of a relation between inflation and age structure on annual OECD data covering 1960–1994 for 20 countries suggest the existence of an age pattern of inflation effects. It is consistent with the hypothesis that increases in the population of net savers dampen inflation, whereas especially the younger retirees fan inflation as they start consuming out of accumulated pension claims. The importance of demographic projections is also documented in Gozluklu (2010), where the role of time variation in the age structure in shaping the long-run co-movement between inflation and financial markets is analyzed.

Our specification does not impose strong priors on the relative importance of demographics to determine equilibrium real rate and the inflation target as we use MY to model the sum of these two variables. In a related paper, McMillan and Baesel (1988) analyze the forecasting ability of a slightly different demographic variable, prime savers over the rest of the population (except under 15) on the interest rates concentrating exclusively on the real interest rate channel.

The idea of using demographics to determine the central bank anchor for policy rates and hence the persistent component of the whole term structure complements the existing literature that uses demography as an important variable to determine the long-run behavior of financial markets (Abel, 2001). While the literature agrees on the life-cycle hypothesis[2] as a valid starting point, there is disagreement on the correct empirical specification and thus the magnitude of demographic effects (Poterba, 2001; Goyal, 2004).

Bakshi and Chen (1994) spur research on the topic by developing the life-cycle risk aversion hypothesis which asserts that individuals become more risk averse as they age. Hence the authors proposed the average age as the demographic variable that drives risk premia. Naturally, the major focus shifted to equity markets and asset allocation and their hypothesis has been tested in several studies in international context (Erb, Harvey, and Viskanta, 1996, Brooks 2000, Ang and Maddaloni, 2005). Our paper differs from the earlier studies in this literature, as our framework is consistent with a strong demographic effects both on equity and bond markets even in a risk-neutral world.

In fact, abundant evidence is available on the impact of the demographic structure of the population on long-run stock-market returns (Ang and Maddaloni (2005), Bakshi and Chen (1994), Goyal (2004), Della Vigna and Pollet (2007)). To our knowledge the study of the empirical relation between demographics and the bond market is much more limited, despite the strong

[2] Life cycle investment hypothesis suggests that agents should borrow when young, invest for retirement when middle-aged, and live off their investment once they are retired.

interest for co-movements between the stock and the bond markets (Lander et al.(1997), Campbell and Vuoltenaho (2004), Bekaert and Engstrom (2010)). Demographics has the potential of explaining comovement between bond and stocks in terms of the relevance of the age structure of population in determining the persistent component of the dividend-price ratio (Favero, Gozluklu and Tamoni (2011)) and interest rates.
Our approach to monetary policy rule has an important difference from the one adopted in the monetary policy literature. In this literature monetary policy has been described by empirical rules in which the policy rate fluctuates around a constant long-run equilibrium rate as the central bank reacts to deviations of inflation from a target and to a measure of economic activity usually represented by the output gap. The informational and operational lags that affects monetary policy (Svensson(1997)), together with the objective of relying upon a robust mechanism to achieve macroeconomic stability (Evans and Honkapohja(2003)), justify a reaction of current monetary policy to future expected values of macroeconomic targets. As the output-gap and the inflation-gap are stationary variables, this framework is not per se capable of accommodating the presence of the persistent component in policy rates. One outstanding empirical feature of estimated instruments rule is the high degree of monetary policy gradualism, as measured by the persistence of policy rates and their slow adjustment to the equilibrium values determined by the monetary policy targets (Clarida et al (2000) and Woodford(2003)). Rudebusch(2002) and Soderlind et al.(2005) have argued that the degree of policy inertia delivered by the estimation of Taylor-type rules is heavily upward biased. In fact, the estimated degree of persistence would imply a large amount of forecastable variation in monetary policy rates at horizons of more than a quarter, a prediction that is clearly contradicted by the empirical evidence from the term structure of interest rates.[3] Rudebusch(2002) relates the "illusion" of monetary policy inertia to the possibility that estimated policy rules reflect some persistent shocks that central banks face. The introduction of demographics allows to model the persistent component of the policy rate as the time-varying equilibrium interest rate is determined by the age-structure of the population.

To map the dynamics of the one-period rate, as determined by the Taylor rule, into that of the yields at longer maturity a model is needed to project future short rates and the risk premium. To this end, the simplest framework is an Expectation Theory model in which term premia are considered as a constant plus noise and some factors are used to project the inflation and the output gaps. Alternatively term premia can be explicitly modeled by estimating affine no-arbitrage term structure models with observable and unobservable factors (Ang and Piazzesi (2003), Bekaert, Cho, and Moreno (2003), Gallmeyer, Hollifield, and Zin (2005), Hordahl, Tristani, and Vestin (2006), Diebold, Rudebusch, and Aruoba (2005), Rudebusch and Wu (2008)),

[3] In a nutshell, high policy inertia should determine high predictability of the short-term interest rates, even after controlling for macroeconomic uncertainty related to the determinants of the central bank reaction function. This is not in line with the empirical evidence based on forward rates, future rates (in particular federal funds futures) and VAR models.

provides a the natural complement to the Taylor rule. In these specifications given the dynamics of the short term rate, a stationary VAR representation for the factors is used to project the entire term structure. The risk premia, when modelled, are identified by posing a linear (affine) relation between the price of risk and the factors. In this case the no-arbitrage assumption allows to pin down the dynamics of the entire term structure by imposing a cross-equation restrictions structure between the coefficients of the state model (the VAR for the factors) and the measurement equations that maps the factors in the yields at different maturities. Ang, Dong and Piazzesi (2005), and Dewachter and Lyrio (2006), investigate how no-arbitrage restrictions can help estimate different policy rules. The potential problem with this general structure is that while yields contain a persistent component, the state evolves as a stationary VAR which is designed to model a mean-reverting process and cannot capture the time series behavior of persistent variables. This discrepancy might therefore explain the, somewhat disappointingly, mixed results from the forecasting performance of affine term structure models (Duffee (2002), Favero, Niu and Sala (2011)). Forecasting interest rates in the presence of a highly persistent component in rates requires the existence of a factor capable of modelling the persistence. We argue this results is achieved by augmenting the traditional set of variables in term structure models with a new factor capturing the age composition of the population as it determines the equilibrium policy rate in a Taylor rule.

3 A Simple Model of the Yield Curve

We motivate our analysis with a simple framework, in which the yield to maturity of the 1-period bond, $y_{t,t+1}$, is determined by the action of the monetary policy maker and all the other yields on n-period (zero-coupon) bonds can be expressed as the sum average expected future short rates and the term premium, $rpy_t^{(n)}$:

$$y_{t,t+n} = \frac{1}{n}\sum_{i=0}^{n-1} E_t[y_{t+i,t+i+1} \mid I_t] + rpy_t^{(n)} \tag{1}$$

$$y_{t,t+1} = rr_t^* + \pi_t^* + \beta(E_t\pi_{t,k} - \pi^*) + \gamma E_t x_{t,q} + u_{1,t+1}$$

In setting the policy rates, the Fed reacts to variables at different frequencies. At the high frequency the policy maker reacts to cyclical swings reflected in the output gap,$x_{t,q}$, i.e. transitory discrepancies of output from its potential level, and in deviation of inflation, $\pi_{t,k}$, from the implicit target of the monetary authority. Monetary policy shocks, $u_{1,t+1}$,also happen. As monetary policy impacts on macroeconomic variable with lags, the relevant variables to determine the current policy rate are k-periods ahead expected inflation and q period ahead expected output gap. However, cyclical swings is not all that matter to set policy rates: we posit that the monetary policy maker watches also slow-moving changes in the economy that take place at a generational frequency, i.e. those spanning several decades, and we relate this to the age structure of population, MY_t as it determines savings behavior and the preferences for inflation. By applying the permanent transitory decomposition to the 1-period policy rates we have:

$$\begin{aligned} y_{t,t+1} &= \rho_0 + \rho_1 MY_t + \rho_2 X_t \\ \rho_0 + \rho_1 MY_t &= rr_t^* + \pi_t^* \\ \rho_2 X_t &= \beta(E_t \pi_{t,k} - \pi^*) + \gamma E_t x_{t,q} + u_{1,t+1} \end{aligned} \tag{2}$$

and, assuming that the inflation gap and the output gap can be represented as a stationary VAR process, yields at longer maturity can be written as follows

$$\begin{aligned} y_{t,t+n} &= \rho_0 + \frac{1}{n}\sum_{i=0}^{n-1} \rho_1 MY_{t+i} + b^{(n)} X_t + rpy_t^{(n)} \\ y_{t,t+n} &= P_{t,t+n} + C_{t,t+n} \\ P_{t,t+n} &= \rho_0 + \frac{1}{n}\sum_{i=0}^{n-1} \rho_1 MY_{t+i} \\ C_{t,t+n} &= b^{(n)} X_t + rpy_t^{(n)} \end{aligned} \tag{3}$$

The decomposition of yields to maturity in a persistent component, reflecting demographics, and a cyclical components reflecting macroeconomic fluctuations and the risk premia, is intuitively consistent with the stylized facts reported in Figures 1-2 in the introductions, namely the presence of a slow moving component related to the demographic variable and common to the entire term structure. Moreover, the relation between the permanent component and the demographic variable is especially appealing for forecasting purposes as the demographic variable is exogenous and highly predictable even for very long-horizons. No statistical model for MY_{t+i} is needed to make the simple model operational for forecasting, as the bureau of Census projections for this strongly exogenous variable can be readily used.

4 Demographics and the Permanent Component of Spot Rates

The existence of a permanent component in spot rates has been identified in the empirical literature by showing that predictors for return based on forward rates capture the risk premium and the business cycle variations in short rate expectations. Fama (2006) explains the evidence that forward rates forecast future spot rates in terms of a mean reversion of spot rates towards a non-stationary long-term mean, measured by a backward moving average of spot rates . Cieslak-

Povala(2011) explain the standard return predictor based on the tent-shape function of forward rates proposed by Cochrane-Piazzesi (2005) as a special case of a forecasting factor constructed from the deviation of yields from their persistent component, as measured by a discounted moving-average of past realized core inflation.

In this section we consider monthly data and we use the framework proposed by Fama(2006) to assess the capability of MY to capture the permanent component of spot rates against that of the different proxies used by Fama(2006) and Cieslak and Povala(2011). This framework is designed to evaluate the forecasting ability for future spot rates of deviations of the spot rates from their long term expected value against that of the spread between forward and spot rates. We implement it by taking three different measures of the permanent component: our proposed measure based on the age composition of population, the measure adopted by Fama based on a moving average of spot rates, and the measure proposed by Cieslak-Povala based on a discounted moving average of past realized core inflation.

Given the decomposition of the spot interest rates, $y_{t,t+n}$[4] in two processes: a long term expected value $P_t, t+n$, that is subject to permanent shocks, and a mean reverting component $C_t, t+n$:

$$y_{t,t+n} = C_{t,t+n} + P_{t,t+n}$$

The following models are estimated

$$y_{t+12x,t+12x+12} \text{-} y_{t,t+12} = a^x + b^x D_t + c^x [f_{t,t+12x,t+12x+12} \text{-} y_{t,t+12}] + d^x [y_{t,t+12} \text{-} P^i_{t,t+12}] + \varepsilon_{t+12x}$$

$$P^1_{t,t+12} = \frac{1}{60} \sum_{i=1}^{60} y_{t-i-1,t+12-i-1} \quad (4)$$

$$P^2_{t,t+12} = \frac{\sum_{i=1}^{120} \upsilon^{i-1} \pi_{t-i-1}}{\sum_{i=1}^{120} \upsilon^{i-1}} \quad (5)$$

$$P^3_{t,t+12} = e^x \frac{1}{12} \sum_{i=1}^{12} MY_{t+i-1} \quad (6)$$

where $f_{t,t+12x,t+12x+12}$ is the one-year forward rate at time t of an investment with settlement after x years and maturity in $x+1$ years, $y_{t,t+12}$ is the one-year spot interest rate, π_t is annual core CPI inflation from time $t-12$ to time t, υ is a gain parameter calibrated at 0.9868 as in Cieslak and Povala, and MY_t is the ratio of middle-aged (40-49) to young (20-29) population in the US, D_t is a step dummy, introduced by Fama in his original study, taking a value of one

[4] We adopt Cochrane and Piazzesi (2005) notation for log bond prices: $p_{t,t+n}$ = log price of n-year discount bond at time t. The continuously compounded spot rate is then $y_{t,t+n} \equiv -\frac{1}{n} p_{t,t+n}$

for the first part of the sample up to August 1981 and zero otherwise. This variable captures the turning point in the behavior of interest rates from a positive upward trend to a negative upward trend occurred in mid 1981 and clearly detectable from Figure 1.
The specification is constructed to evaluate the predictor based on the cyclical component of rates against the forward spot spread. In his original study, Fama found that, conditional on the inclusion of the dummy in the specification, this was indeed the case. This evidence is consistent with the fact the dominant feature in the spot rates of an upward movement from the fifties to mid 1981 and a downward movement from 1981 onwards is not matched by any similar movement in the forward-spot spread which looks like a mean reverting process over the sample 1952-2004. We extend the original results by considering alternative measures of the permanent component over a sample up to the end of 2008[5]. The results from estimation on monthly data are reported in Table 1

Insert Table 1 about here

We consider forecasts at the 2,3,4 and 5-year horizon. For each horizon we estimate first a model with no cyclical component of interest rates but only the forward spot spread, then we include the three different proxies for the cyclical components of interest rates. The estimation of the model with the restriction $d^x = 0$ delivers a positive and significant estimate of c^x with a significance increasing with the horizon x. However, when the restriction $d^x = 0$ is relaxed, then the statistical evidence on the significance of c^x becomes much weaker In fact, this coefficient is never significant when the cycle is specified using the demographic variable to measure the permanent component and much less significant when any measure of the cycle in interest rates is introduced in the specification. The inclusion of the dummy is necessary only in the case of the Fama-cycle, while in the cases of the inflation based cycle and the demographic cycle the inclusion of the dummy variable capturing the turning points in the underlying trend is not necessary anymore, witnessing the capability of demographics and smoothed inflation of capturing the change in the underlying trend affecting spot rates. The performance of the inflation cycle and the demographic cycle is very similar at the 2 and 3-year horizon, while the demographic cycle dominates at the 4 and 5-year horizon. The estimated coefficient on the demographic variable is very stable at all horizons, while the one on the discounted moving average of past inflation is more volatile.
The comparative properties of the three alternative specifications for the permanent componentare further investigated via pairwise encompassing regressions. After specifying three different forecasting model based on the three alternative specifications, pairwise regressions of the target variable on the forecast based on the demographic proxy for the permanent component and on each of the two backward looking estimates have been performed.

[5] 1-year Treasury bond yields are computed from the (monthly and quarterly) price series from the Fama CRSP zero coupon files. Middle-young ratio data is available at annual frequencies from Bureau of Census (BoC) and it has been interpolated to obtain monthly and quarterly series.

In practice, after estimating the three alternative forecasting models at all relevant horizons

$$
\begin{aligned}
(y_{t+12x,t+12x+12} - y_{t,t+12})^{FAMA} &= \hat{\beta}_0 + \hat{\beta}_1 y_{t,t+12} + \hat{\beta}_2 P^1_{t,t+12} + \hat{\beta}_3 D_t \\
(y_{t+12x,t+12x+12} - y_{t,t+12})^{CP} &= \hat{\gamma}_0 + \hat{\gamma}_1 y_{t,t+12} + \hat{\gamma}_2 P^2_{t,t+12} \\
(y_{t+12x,t+12x+12} - y_{t,t+12})^{MY} &= \hat{\delta}_0 + \hat{\delta}_1 y_{t,t+12} + \hat{\delta}_2 P^3_{t,t+12} \\
x &= 2, 3, 4, 5
\end{aligned}
$$

The following encompassing regressions are performed at each horizon:

$$
\begin{aligned}
(y_{t+12x,t+12x+12} - y_{t,t+12}) &= b^1 (y_{t+12x,t+12x+12} - y_{t,t+12})^{MY} + c^1 (y_{t+12x,t+12x+12} - y_{t,t+12})^{FAMA} + u_{t+12x} \\
(y_{t+12x,t+12x+12} - y_{t,t+12}) &= b^2 (y_{t+12x,t+12x+12} - y_{t,t+12})^{MY} + c^2 (y_{t+12x,t+12x+12} - y_{t,t+12})^{CP} + v_{t+12x}
\end{aligned}
$$

The results reported in Table 2, show that the model based on MY is never dominated by the two alternatives and it is overall the best performing model: the demographics specification can explain almost entirely the forecasting errors of the model based on the Cieslak-Povala trend at horizons longer than 3-years and the forecasting errors based on the Fama trend at horizons shorter than 4-years. Note that neither the MY nor the CP estimates of the permanent component are estimated by including the dummy in the specification, while the Fama trend is aided by the inclusion of this variable. Restricting the coefficient $\hat{\beta}_3$to zero makes the Fama trend uniformly dominated by the two alternatives at all forecasting horizons.

This evidence shows that the demographic structure of US population can be at least as useful as (weighted) averages of past inflation and past interest rates to model the permanent component of short-term rates. The empirical model adopted in this section is mutated by Fama (2006) and it is aimed at identifying, within sample, the permanent component of spot rate. The major comparative advantage of the demographic based measure of the permanent component emerges when models are used for out-of-sample predictions as precise predictions of the age-structure of population are available for very long forecasting horizon. Future values of the age structure of the population up to very long forecasting horizon are precisely predicted and made available by the Bureau of Census for up to 40 years ahead. This gives a specific advantage to the specification of the permanent component in yields based on demographics with respect to those based on weighted average of inflation and interest rates. Predictions of the age structure of population over the next ten-years are more precise and reliable than predictions of the trends of inflation and interest rates based on slow moving averages. To exploit this feature of the demographic data we devote the next section to build full-term structure models with a demographic factor and to study their forecasting performance.

5 Term Structure Models with Demographics

In this section we consider two empirical versions of the framework that models the one-period yield via the central bank reaction function and all yields at longer maturities as the sum of future expected policy rates and the term premium.

5.1 An Expectation Theory Model with Demographics

Our first specification is an "operational" version of the simple model for the components of the yield curve discussed in Section 3. We adopt the following specification on quarterly data:

$$
\begin{aligned}
y_{t,t+1} &= (\rho_0 + \rho_1 MY_t + \rho_2 \pi_{t-1} + \rho_3 x_{t-1})(1-\varphi) + \varphi y_{t-1,t} + u_{1,t+1} \qquad (7) \\
\rho_0 + \rho_1 MY_t &= rr_t^* + \pi_t^* \\
\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} &= B_0 + B_1(L) \begin{bmatrix} y_{t-1,t} \\ \pi_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{2,t} \\ u_{3,t} \end{bmatrix} \\
y_{t,t+n} &= y^*_{t,t+n} + k_n + u_{4,t+n} \\
y^*_{t,t+n} &= \frac{1}{n} \sum_{i=0}^{n-1} E_t [y_{t+i,t+i+1} \mid I_t]
\end{aligned}
$$

The one -period (three-month) rate is modelled by augmenting a standard reduced-form Taylor rule with the demographic variable MY_t, which captures the equilibrium level of the short term rate. The Taylor rule is in reduced form as contemporaneous rather then expected output gap and inflation enters the specification, while the true "structural" reaction function should depend on expected output gap and inflation. Output gap and inflation are proxied by using two observable dynamic factors extracted from large-data to mimic the relevant "real" and inflation information which the Fed uses to set the monetary policy rate in a data-rich environment (Bernanke and Boivin (2003), Ang, Dong and Piazzesi (2005). In particular, we consider the two factors estimated by Ludvigson and Ng (2009). The rule includes one-lag of the monetary policy instrument to allow for interest-rate smoothing (see Clarida et al., 2003) of policy rates around their target. The two factors are projected via an augmented VAR that also allows for a feedback between monetary policy and the factors dynamics. Yields at all maturities longer than one period are projected via the weak expectations theory as observed yields are decomposed into the discounted future expected policy rate and a stationary error term that captures the fluctuations of the unobservable term premia $rpy_t^{(n)}$ around their means k_n. We shall evaluate the importance of the inclusion of the demographic variable in the model by conducting two alternative forecasting exercises based on two specifications: an unrestricted version with demographics and a restricted version, obtained by imposing $\rho_1 = 0$, which rules out the impact of demographics from yields at all maturities.

5.1.1 Empirical Results

Our empirical analysis is based on full-sample estimation of the model over the period 1964:2-2007:4 and pseudo out-of-sample forecasting exercise based out-of-sample simulation over the period 1989Q1-2007Q4. The properties of the data are illustrated in Table 3

Insert Table 3 about here

The descriptive statistics reported in Table 3 highlights the persistence of all yields which is not matched by the persistence of the macroeconomic factors extracted from the large data-set and it is instead matched by the persistence of the demographic variable MY. Figure 3 illustrates graphically the point by showing the quick mean reversion of the macroeconomic indicators.

Insert Figure 3 about here

We report the results of the full-sample estimation of the expectations theory model described above in Table 4. The Table reports parameters estimated in the unrestricted version of the model with demographics and the restricted version of the model, that, by imposing $\rho_1 = 0$, replicates standard Taylor rules.

Insert Table 4 about here

Several comments are in order on the results of the full sample estimation.
MY_t is strongly significant in the estimated reaction function, and degree of estimated interest rate smoothing is affected by the inclusion of this variable in the specification. When the restriction $\rho_1 = 0$ is imposed the coefficient on the lagged policy instrument raises from 0.8 to 0.9. Figure 4 illustrates our demographics based estimate of $rr_t^* + \pi_t^*$, the associated 95 per cent confidence interval, and the actual three-month rate. Note that the actual rate fluctuates around the target and the only period in which the actual rate falls consistently outside the confidence interval is from 1979 to 1982. This period coincides with the Non-Borrowed Reserves targeting period, in which the short-term rate was not controlled but it was instead endogenously determined by the target for the quantity of reserves.

Insert Figure 4 about here

The coefficients on the two macroeconomic factors in the policy rule are harder to interpret as, in the presence of lags in the monetary policy transmission mechanism, our empirical specification is a reduced form version of the forward-looking central bank reaction function and no structural interpretation can be attached to the observed impact of the two factors for inflation and the output gap on interest rates. We note, however, that the coefficient describing the long-run response of monetary policy to inflation is consistent with the Taylor principle that prescribes raising real interest rates in presence of an inflationary shocks. The estimate of the term structure of risk premia is upward sloping and ranges from an average of 50 basis point at the 1-year maturity to just over 100 basis points at the 5-year maturity. This shape of the term structure is very similar in the restricted and unrestricted model. However, the fluctuations of the $y^*_{t,t+n}$ constructed on the basis of the two alternative specifications are very different. Figure 5 illustrates this evidence by reporting actual 5-year yields with those obtained by solving forward at each point in time the two alternative specifications to obtain the full path of the 1-period rates up to twenty periods (5-year) ahead and by generating $y^*_{t,t+20}$ as their average plus a constant. The path for the $y^*_{t,t+20}$ generated by the two alternative specification is very different, although the average risk premium is very similar. The model with demographics produces simulated $y^*_{t,t+20}$ that tracks the actual data better than the simulation based on the restricted version ($\rho_1 = 0$) of the specification

Insert Figure 5 here

The second panel of Figure 5 allows to assess the empirical performance of the model based 5 year-3 month spread, $S^*_{t,t+20} = y^*_{t,t+20} - y_{t,t+1}$, in tracking the actual 5 year-3 month spread, $S_{t,t+20} = y_{t,t+20} - y_{t,t+1}$. This is an interesting challenge for the model-based on demographics because the ET model based spread does not contain any permanent component but it is the sum of two stationary components: the weighted sum of future change in policy rates and the risk premium.

In fact,

$$y_{t,t+n} = \frac{1}{n}\sum_{i=0}^{n-1} E_t[y_{t+i,t+i+1} \mid I_t] + k_n + u_{4t}$$

implies that

$$\begin{aligned} S_{t,t+20} &= y_{t,t+20} - y_{t,t+1} \\ &= \sum_{i=1}^{n-1} \frac{n-i}{n} E_t[\Delta y_{t+i,t+i+1} \mid I_t] + k_n + u_{4t} \\ &= S^*_{t,t+20} + k_n + u_{4t} \end{aligned} \tag{8}$$

So the comparison between $S^*_{t,t+20}$ and $S_{t,t+20}$ allows to evaluate which portion of the total variation of the 5 year-3month spread can be attributed to the change in the permanent component of interest rates, as captured by demographics. Figure 5 illustrates a rather remarkable performance of the $\left(S^*_{t,t+20} + k_n\right)$ based on demographics in capturing the fluctuations of $S_{t,t+20}$. In fact, changes in the policy rates predicted by the model based on demographics play a major role in the determining the term spread and clearly dominate the prediction of the specification based on macroeconomic factors only. The main fluctuations in the risk premium that add some explanatory power to demographics- (weak) ET predicted spread for predicting the actual spread seems to be a break in its mean at the beginning of the eighties.

Overall, the within sample analysis offers some interesting evidence in support of the hypothesis that demographics is an important determinant of the equilibrium policy rate in the central bank reaction function. The presence of a common demographics related component can also explain the common persistent component in the term structure. However, the main advantage of the specification based on demographics should emerge in simulation and out-of-sample forecasting exercise where the predictability of demographic trends can be fully exploited to track the future evolution of the equilibrium policy rate. Figure 6 illustrate the importance of introducing the demographic variables when the model is used in simulation.

Insert Figure 6 here

Given full-sample estimation the two version of the model are simulated dynamically from the first observation onward to generate yields at all maturities. The results show that, while the model without demographics converges to the sample mean, the model with demographics feature projections that have fluctuations consistent with those of the observed yields also for very long forecasting horizons.

We turn now to the out-of-sample forecasting performance. Out-of-sample forecasts for the yield curve based on rolling estimation (we consider a rolling window of one hundred observations) of the two versions of the expectation theory model. The initialization sample is 1964Q1-1988Q4. We report in Tables 5-6 the analysis of the forecasting performance of the alternative models.

Insert Table 5-6 here

Table 5 reports measure of the comparative forecasting performance based on the ratio of the root mean squared forecast error (RMSFE) of the model with demographics to the RMSFE of the model without demographics and a random walk forecast . Bold characters are used when the ratio of the model's RMSFE to that of the alternative benchmarks is in the range [0.9, 1) while bold and underline characters are used for rations smaller than 0.9. We also report in parentheses the p-values of the forecasting accuracy test proposed by Giacomini and White (2006). A p-value below 0.01 (0.05, 0.10) indicates a significant difference in forecasting performance at the 1% (5%, 10%) level. Table 6 complements the information of Table 5 by reporting the out-of-sample R^2 statistics (Campbell and Thomson, 2008) which is computed as

$$\mathrm{R}^2_{OS,x} = 1 - \frac{\sum_{t=t_0}^{T} \left[\left(y_{t+12x,t+12x+12} - y_{t,t+12} \right) - \left(y_{t+12x,t+12x+12} - y_{t,t+12} \right)^{pr} \right]^2}{\sum_{t=t_0}^{T} \left[\left(y_{t+12x,t+12x+12} - y_{t,t+12} \right) - \left(y_{t+12x,t+12x+12} - y_{t,t+12} \right)^{av} \right]^2}$$

where $(y_{t+12x,t+12x+12}$ -$y_{t,t+12}$ $)^{pr}$ are the model-based projections at horizon x and $(y_{t+12x,t+12x+12}$ -$y_{t,t+12}$ $)^{av}$ is the historical average change computed with data available at the time of forecasting. In our exercise, $t_0 = 1994$ and T= 2008. The R^2_{OS} is positive when the predictive regression has a lower mean square error than the prevailing historical mean. The evidence reported in the two tables strongly supports the hypothesis that the demographics based model dominates all alternatives, with some exceptions at the very short forecasting horizon, where the comparative advantage of the demographic model cannot be exploited.

The statistical evidence in Table 5-6 is more easily interpreted by visual inspections of the different out-of sample forecasts that we report in Figure 6

Insert Figure 7 here

The out-of-sample forecast from the two alternative models do differ rather importantly and the demographics factor helps in generating long-horizon forecast for yields that are closer to the ex-post observed one and clearly dominate those based on the model without demographics in their capability of matching fluctuations in the observed data.

5.2 An Affine Term Structure Model with Demographics

The model in the previous section consider the term premium as a residual factor that is obtained as the difference between observed yields and yields consistent with the expectations theory. In this section we consider the role of demographics within a more structured specification that explicitly models term premia. In particular, we estimate the following affine term structure model:

$$
\begin{aligned}
y_{t,t+n} &= -\frac{1}{n}\left(A_n + B_n' X_t + \Gamma_n \mathrm{MY}_t^n\right) + \varepsilon_{t,t+1} \qquad \varepsilon_{t,t+n} \sim N(0,\sigma_n^2) \\
y_{t,t+1} &= \delta_0 + \delta_1' X_t + \delta_2 \mathrm{MY}_t \\
X_t &= \mu + \Phi X_{t-1} + \nu_t \quad \nu_t \sim i.i.d.N(0,\Omega)
\end{aligned}
$$

where $\Gamma_n = \left[\gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n\right]$, and $\mathrm{MY}_t^n = [\mathrm{MY}_t, \mathrm{MY}_{t+1} \cdots, \mathrm{MY}_{t+n-1}]'$, $y_{t,t+n}$ denotes the yield at time t of a zero-coupon government bond maturing at time $t+n$, the vector of the states $X_t = [f_t^o, f_t^u]$, where $f_t^o = [f_t^\pi, f_t^x]$ are two observable factors extracted from large-data sets to project the inflation and output gap using all relevant "real" and inflation information which the Fed uses to set the monetary policy rate in a data-rich environment (Bernanke and Boivin (2003), Ang, Dong and Piazzesi (2005), while $f_t^u = \left[f_t^{u,1}, f_t^{u,2}, f_t^{u,3}\right]$ contain unobservable factor(s) capturing fluctuations in the unobservable interest rate target of the Fed orthogonal to the demographics fluctuations, or interest rate-smoothing in the monetary policy maker behavior. Consistently with the previous section, we consider the two factors estimated by Ludvigson and Ng (2009) to capture "real" and inflation information.

Our specification for the one period-yield is a generalized Taylor rule in which the long-term equilibrium rate is related to the demographic structure of the population, while the cyclical fluctuations are mainly driven by the output gap and fluctuations of inflation around the implicit central bank target. Note that in our specification the permanent component of the 1-period rate is modelled via the demographic variable and the vector of the states X_t is used to capture only cyclical fluctuations in interest rates. Hence, it is very natural to use a stationary VAR representation for the states that allows to generate long-term forecasts for the factors and to map them into yields forecasts. MY_t is not included in the VAR as reliable forecasts for this exogenous variable up to very long-horizon are promptly available from the Bureau of Census. The model is completed by assuming a linear (affine) relation between the price of risk, Λ_t,and the states X_t by specifying the pricing kernel, m_{t+1}, consistently and by imposing no-arbitrage restrictions[6] (see, for example, Duffie and Kan (1996), Ang and Piazzesi (2003)):

$$\Lambda_t = \lambda_0 + \lambda_1 X_t$$

$$m_{t+1} = \exp(-y_{t,t+1} - \tfrac{1}{2}\Lambda_t' \Omega \Lambda_t - \Lambda_t \varepsilon_{t+1})$$

$$A_{n+1} = A_n + B_n' \left(\mu - \Omega \lambda_0\right) + \tfrac{1}{2} B_n' \Omega B_n + A_1$$

$$B_{n+1}' = B_n' \left(\Phi - \Omega \lambda_1\right) + B_1'$$

$$\Gamma_{n+1} = [-\delta_2, \Gamma_n]$$

$$\lambda_0 = \begin{bmatrix} \lambda_0^{\pi} \\ \lambda_0^{x} \\ \lambda_0^{u,1} \\ \lambda_0^{u,2} \\ \lambda_0^{u,3} \end{bmatrix} \quad \lambda_1 = \begin{bmatrix} \lambda_1^{\pi} & & \cdots & & \mathbf{0} \\ & \lambda_1^{x} & & & \\ \vdots & & \lambda_1^{u,1} & & \vdots \\ & & & \lambda_1^{u,2} & \\ \mathbf{0} & & \cdots & & \lambda_1^{u,3} \end{bmatrix}$$

Note that the imposition of no-arbitrage restrictions allows to model the impact of current and future demographic variables on the term structure in a very parsimonious way, as all the effects on the term structure of demographics depend exclusively on one parameter: δ_2. Our structure encompasses traditional affine term structure model with macroeconomic factors and no demographic variable as this specification is obtained by setting $\delta_2 = 0$. The no arbitrage restrictions guarantees that when $\delta_2 = 0$ also $\Gamma_n = 0$: as demographics enter the specification of yields at longer maturities only via the expected one-period yield, the dynamics of yields at all maturities become independent from demographics if MY_t does not affect the one-period policy rate. However, when the restriction $\delta_2 = 0$ is imposed, the structure faces the problem highlighted in the previous section of having no structural framework for capturing the persistence in policy rates. In fact, to match persistence in the policy rates, some of the unobservable factors must be persistent as the observable factors are, by construction, stationary. Then, the

[6] See the Appendix for a formal derivation of the restrictions

VAR for the state will include a persistent component which will make the long-term forecasts of policy rates, necessary to model the long-end of the yield curve, highly uncertain and unreliable. In the limit case of a non-stationary VAR long-term forecast become useless as the model is non-mean reverting and the asymptotic variance diverges to infinite.

5.2.1 Empirical Results

As in the previous section, we estimate the model on quarterly data by considering the 3-month rate as the policy rate.

And again we consider two alternative specifications: we evaluate the performance of our specification with MY against that of a benchmark discrete-time no-arbitrage term structure model obtained by imposing the restriction $\delta_2 = 0$ on our specification. Following the specification analysis of Pericoli and Taboga (2008), we focus on a parsimonious model including three latent factors and only contemporaneous values of the macro variables. We use the Chen and Scott's (1993) methodology; given the set of parameters and observed yields latent variables are extracted by assuming that number of bonds which are priced exactly is equal to the number of unobserved variables. Hence we assume that 3-month, 2-year and 5-year bond prices are measured without error and estimate the model with maximum likelihood. We assume the state dynamics to follow a VAR(1). We impose the following restrictions on our estimation (Favero, Niu and Sala, 2010):

i) the covariance matrix Ω is block diagonal with the block corresponding to the unobservable yield factor being identity, and the block corresponding to the observable factors being unrestricted, i.e.

$$\Omega = \begin{bmatrix} \Omega^o & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

ii) the loadings on the factors in the short rate equation are positive, $0 \leq$ -A_1

iii) $f_0^u = 0$.

The results from estimation on the full sample 1954-2007 are reported in Table 7a and 7b.

Insert Table 7 here

The results show a weaker evidence for the significance of demographics in the reaction function than in the case of the weak Expectations Theory model. However, the estimated dynamics of the unobservable factors is very different when the benchmark model is augmented with MY. In fact, in the benchmark macro-finance model without demographics factors are very persistent and the matrix $(\Phi - I)$ describing the long-run properties of the system is very close to be singular, while this near singularity disappears when the persistent component of yields at all maturities is captured by the appropriate sum of current and future age structure of the population. In this case the VAR model for the states becomes clearly stationary and long-term predictions are more precise and reliable.

We complement the results of full sample estimation by analyzing also the properties of out-of-sample forecasts of our model at different horizons. In our multi-period ahead forecast, we choose iterated forecast procedure, where multiple step ahead states are obtained by iterating the one-step model forward

$$\widehat{y}_{t+h,t+h+n|t} = a_n + b_n \hat{X}_{t+h|t} + \Gamma_n \mathrm{MY}_{t+h}^n$$
$$\hat{X}_{t+h|t} = \sum_{i=0}^{h} \hat{\Phi}^i \mu + \hat{\Phi}^h \hat{X}_t$$

where $a_n = -\frac{1}{n} A_n, b_n = -\frac{1}{n} B_n$ are obtained by no-arbitrage restrictions. Forecast are produced on the basis of rolling estimation with a rolling window of one hundred observation, the first sample used for estimation is 1964Q1-1988Q4. We consider 6 forecasting horizons (denoted by h): one quarter, one year, two years, three years, four years and five years. For the one quarter ahead forecasting horizon, we conduct our exercise for all dates in the period 1989Q1 - 2007Q4, a total of 76 periods; for the 1-year ahead forecast, we end up with a total of 73 forecasts, and so on, up to the 5-year ahead forecast, for which we end up with 57 forecasts.

Forecasting performance is measured by the ratio of the root mean squared forecast error (RMSFE) of the affine model with demographics to the RMSFE of a random walk forecast and to the RMSFE of the benchmark yield-macro model without the demographic variable. Forecasting results from different models are reported in Table 8-9.

Insert Table 8-9 here

Table 8 reports the RMSFE of the model with demographics relative to the random walk and benchmark yield-macro model $(\delta_2 = 0)$.Table 9 complements the information of Table 9 by reporting the out-of-sample R^2 statistics (Campbell and Thomson, 2008). The evidence shows that the forecasting performance of the affine model with demographics dominates the traditional affine term structure model. Comparison of Tables 8-9 with Tables 5-6 shows that the best predictive performance is obtained by the (weak) ET-model with demographics. While including demographics in term structure models seems decisive to generate a better forecasting performance, using an affine structure to model a time-varying risk does not improve on the forecasting performance of a simpler model that does not impose no-arbitrage restrictions, uses demographics to project future policy rates and considers the term premia as constant over time.

6 Long-Term Projections

One of the appealing features of an affine term structure model with demographics factors is that the availability of long-term projections for the age-structure of the population which can be exploited to produce long-term projections for the yield curve. In our specification yields at time $t+j$ with maturities $t+j+n$ are functions of all realization of MY between $t+j$ and $t+j+n$. The exogeneity of the demographic variable and the availability of long term projections is combined in the affine model with a parsimonious parameterization generated by the no-arbitrage restrictions that allow to weight properly all future values of MY with the estimation of few coefficients. As a result future paths up to 2045 can be generated for the entire term structure, given the availability of demographic projections up to 2050. Note that an affine model with demographics allow for a very different out-of-sample dynamics from those that can be generated by any model based on the decomposition of interest rates in long-term expected mean and mean-reverting component and on the measurement of the long-term expected mean as the moving average of the most recent past five years of the spot rates. Consider, for example, long-term projections from the Fama model. They would not differ from those of an autoregressive process, they would just converge more slowly to the unconditional mean. To illustrate this feature of the model with demographics report in Figure 9 all yields reported in Figure 1 by also drawing their predicted out-of-sample path up to 2045, based on the best the weak ET model .

Insert Figure 8 here

The predicted path of the age structure of the population drives the forecast of the term structure up in the range between six per cent and eight per cent over the next twenty years. Any model of the term structure with stationary macro factors estimated over the last twenty years could never produce such forecasts, as they are clearly far away from the unconditional sample mean.

7 Demographics and the Permanent Component of Inflation

The evidence discussed so far illustrates that the target long-run interest rates in a Taylor rule could be modelled as a function of the age composition of population and that embedding the rule within with a (weak) Expectations Theory model generates a superior performance for forecasting the entire term structure, especially at long-horizons. The target interest rate in a Taylor rule can be decomposed in a target inflation rate and a target real interest rate. If demographics explain the permanent component of rates, than it is more natural to relate the age structure of population to the target inflation rates as the general prior is that real interest rates are clearly less persistent than inflation rates. It is therefore interesting to check our evidence in the previous sections by investigating the relationship between demographics and the long-run component of inflation. To this end we have a natural benchmark in the Cieslak-Povala proxy for the permanent component of interest rates, which is explicitly based on the permanent component of inflation. The two panels of Figure 9 report the permanent and the cyclical component of inflation based on three different measures: the long-run equilibrium rate estimated form our Taylor-rule with demographics ($13.26 - 0.091 * MY_t$), the Cieslak-Povala permanent component, and an Hodrick-Prescott filter of inflation. The Figure shows a strong comovement of the three alternative components that generate cycles with the same peaks and throughs.

Insert Figure 9 here

To further investigate the visual evidence, we estimate two alternative dynamic specifications for annual core CPI inflation on our sample of quarterly data:

$$\pi_t = \beta_0 + \beta_1 \pi_{t-1} + (1 - \beta_1)(\pi_t^*) + \varepsilon_t$$
$$\pi_t^* = \alpha_0 + \alpha_1 MY_t$$
$$\pi_t^* = \pi_{t-1}^* + (1 - 0.96)\left(\pi_t - \pi_{t-1}^*\right)$$

In the first specification the permanent component of inflation is captured by the age-structure of population, while the alternative specification is based on the Cieslak-Povala trend. Results from estimation are reported in Table 10, which also contains the results from the estimation of the a simple dynamic model for the nominal interest rates that includes only demographics as an explanatory variable:

$$y_{t,t+1} = (\rho_0 + \rho_1 MY_t)(1 - \varphi) + \varphi y_{t-1,t} + v_t$$

The estimate of the coefficient on demographics in the specifications for inflation and the nominal rate is very similar and strongly significant. The evidence that demographics performs slightly better in modelling nominal rates than in modelling inflation leaves some role for demographics in explaining fluctuations of real short-term rates . The within sample fit of the dynamic model for inflation based on a demographic driven permanent is at least equivalent to that based on the Cieslak-Povala permanent component.

The true comparative advantage of the demographics based trend emerges in out-of-sample projections. Figure 10 shows out-of-sample forecasts for inflation, from 1990 onwards, for an increasing horizon.

Insert Figure 10 here

In particular, we report the projections for annual inflation based on the model with demographics and on the Cieslak-Povala trend for an increasing horizon going from 1-year to 2-year and for all horizons from the one-quarter ahead to the 20-year ahead. The results speak clearly in favour of the demographic based projections, that have he capability of modelling an out-of-sample mean of inflation different from the mean within sample. Further interesting evidence on the capability of the demographics based model to predict inflation is provided by Figure 11 which (favourably) compares the one-year ahead prediction for core inflation based on the demographics model with two survey based predictions (the Livingston Survey and the survey of Professional Forecasters).

Insert Figure 11 here

8 Robustness

This section examines the robustness of our results along four dimensions. First, we consider the role of MY forecasting excess returns in the standard regression framework originally introduced by Cochrane and Piazzesi(2005). Second, in all forward projections we have implemented so far we have treated MY_{t+i} at all relevant future horizons as a known variable. Predicting MY requires projecting population in the age brackets 20-29, and 40-49. Although these are certainly not the age ranges of population more difficult to predict (improvement in mortality rates that have generated over the last forty years difference between actual population and projected population are mostly concentrated in older ages, after 65) the question on the uncertainty surrounding projections for MY is certainly legitimate. Third, one might object that our statistical evidence on MY_t and the permanent component of interest rates is generated by the observation of a couple of similar paths of nonstationary random variables. Although the spurious regression problem is typical in static regression and all the evidence reported so far is based on estimation of dynamic time-series model, some simulation based evidence might be helpful to strengthen our empirical evidence. Fourth, all the empirical results reported so far are based on US data, international evidence can therefore be used to produce further evidence based on a different data-set.

8.1 Forecasting Excess Returns with MY

Consider forecasting, at t, the log return to a five-year bond from t to t + 12 less the log return to a one-year bond. At t + 12, the bond will be a four-year bond. (This is a standard regression since Cochrane and Piazzesi(2005)). According to our model, cyclical deviations of yields from their permanent component, captured by demographics, reflect either shifts in expectations of future monetary policy or risk premia. Independently form the nature of the cycle in yields, time-t deviations of yields from their persistent component should contain incremental information for forecasting the future log excess returns .

Following Cochrane and Piazzesi (2005) define the log holding period return from buying an n-year zero-coupon bond at time t and selling it as an $n-1$ year bond at time $t+1$ as

$$r^n_{t,t+1} = p_{t+1,t+1+(n-1)} - p_{t,t+n}$$

Denote excess log returns by

$$xr^n_{t,t+1} \equiv r^n_{t,t+1} - y_{t,t+1}$$

for $n = 2, 3, 4, 5$.

Then the following regression model is estimated over the sample 1952:6 through 2008:12 with monthly data

$$xr^n_{t,t+1} = b_0 + b_1^{'}\mathbf{y}_t + b_2 MY_t + \varepsilon^n_{t,t+1}$$

where $\mathbf{y}_t$ contains yields for maturities of one through five years. The inclusion among the regressor of the level of the yields and MY_t allows to interpret the model as an Error Correction specification where deviation of the yields from their permanent component are used to predict excess returns.[7]

Insert Table 11 here

The results of the estimation, reported in Table 11, shows the presence of predictable variation in bond excess returns on the basis of the cyclical component of yields. Importantly, if the coefficient $b_1^{'}$ is restricted to zero also the coefficient b_2 becomes statistically insignificant. This evidence is consistent with the assumption made in our term structure specification that the population ratio does not change risk premia on bonds, and it is also not suprising, as risk premia are not particularly persistent, while the population ratio is instead highly persistent.

8.2 The Uncertainty on Future MY

To analyze the uncertainty on projections on MY we use the evidence produced by the Bureau of Census 1975 population report, which published projections of future population by age in the United States from 1975 to 2050, reporting annual forecast from 1975 to 2000 and five-year forecasts from 2000 to 2050. The report contains projections based on three different scenarios for fertility, which is kept constant and set to 1.7, 2.1 and 2.7, respectively. All three scenarios are based on the estimated July 1, 1974 population and assume a slight reduction in future mortality and an annual net immigration of 400,000 per year. They differ only in their assumptions about "future fertility". Since there is only 5-year forecasts from 2000-2050, we interpolate 5-year results to obtain the annual series. Then we construct MY_t ratio by using this annually projection results of different fertility rates from 1975 to 2050.

[7] Note that this specification is the equivalent for the bond-market of the specification adopted for the stock market by Favero et al.(2010) who successfully use deviations of the log price dividend from its permanent component modelled by MY_t to predict stock market returns.

To evaluate the uncertainty surrounding projections for our relevant demographic variable, Figure 12 reports plot actual MY_t and projected MY_t in 1975.

Insert Figure 12 here

The actual annual series of MY_t is constructed based on information released by BoC until December, 2010, while, for the period 2011 to 2050 we use projections contained in the 2008 population report. The figure illustrates that the projections based on the central value of the fertility rate virtually overlaps with the observed data up to 2010 and with the later projections for the period 2011-2050 (Davis and Li (2005)). The impact on MY of different assumptions on fertility is also rather modest.

8.3 A Simulation Experiment

To assess the robustness of our results we started from the estimation of a simple autoregressive model for 3-month rates over the full sample. By bootstrapping the estimated residuals we have then constructed one thousand artificial time series for the short-rates. These series are very persistent (based on an estimated AR coefficient of .93) and generated under the null of no-significance of MY in explaining the 3-month rates. We have then run one thousand regression by augmenting an autoregressive model for the artificial series with MY_t. Figure 13 shows that the probability of observing a t-stat of -3.96 on the coefficient on MY_t is 0.01 (the t-stat on MY_t in the actual regression of the 3-month rate, its own lags and the demographic variable)

Insert Figure 13 here

This fraction extremely close to zero of simulated t-stat capable of replicating the observed results provides clear evidence against the hypothesis that our statistical results on demographics and the permanent component of interest rates are spurious.

8.4 International Evidence

We consider some international evidence to evaluate on a larger and different data-set the evidence reported so far. In particular, the demographic variable MY_t is constructed for a large panel of 40 countries over the period 1961-2009 (unbalanced panel)[8]. We consider the performance of augmenting autoregressive models for nominal bond yields and continuously compounded annual inflation[9] against the benchmark where the effects of demographics is restricted to zero.

The results from the estimation are reported in Table 12.

[8] The results are robust when we contruct a smaller panel with balanced data. The demographic data is collected from Worldbank database.

[9] Bond yield and inflation data is collected from Global Financial data. Long term bond yields are 10-year yields for most of the countries, except Japan (7-year), Finland, South Korea, Singapore (5-year), Mexico(3-year), Hong Kong(2-year).

Insert Table 12 here

The evidence on the importance of MY in capturing the persistent component of inflation and nominal yields is confirmed by the panel estimation. Note that not only the coefficient on MY is significant in both the specification for inflation and the nominal yields, but also the similarity of the two coefficients that emerged from US data is confirmed in the international panel.

9 Conclusions

The entire term structure of interest rates features a common persistent component. Our evidence has shown that it is possible to model such a persistent component using a demographic variable, to ratio of middle-aged to young population, MY_t. The age composition of the population defines the persistent component in 1-period yields as it determines the equilibrium nominal rate in the central bank reaction function. Fluctuations in demographics are then reflected in the whole term structure via the expected policy rates components in the yields at maturity longer than 1-year. Our specification does not impose strong priors on the relative importance of demographics to determine equilibrium real rate and the inflation target, as we use MY_t to model the sum of these two variables. The empirical evidence shows that there is a strong effect of demographics on the persistent component of inflation. The relation between the persistent component of inflation and the age structure of population explains almost entirely the relation between the persistent component of nominal rates and demographics. The presence of demographics in short-term rates allows more precise forecast of future policy rates, especially at very long-horizon, and helps modelling the entire term structure. Term structure macro-finance models with demographics clearly dominate traditional term-structure macro-finance models. When demographics are entered among the determinants of short-term rates, a simple model based on a Taylor rule and a weak expectations theory specifications for yields at longer maturities outperforms in forecasting traditional term structure models. There is a simple intuitive explanation for this results: traditional Taylor-rules and macro finance model do not include an observed determinant of yields capable of capturing their persistence. Linking the long-term central bank target for interest rates to demographics allows for the presence of a slowly moving target for policy rates that fits successfully the permanent component observed in the data. Rudebusch(2002) relates the "illusion" of monetary policy inertia to the possibility that estimated policy rules reflect some persistent shocks that central banks face. Our evidence illustrates that such persistent component is effectively modeled by the age structure of the population. The successful fit is then associated to successful out-of-sample predictions because the main driver of the permanent component in spot rates is exogenous and predictable. Overall, our results show the importance of including the age-structure of population in macro-finance models. As pointed out by Bloom et al.(2003) one of the remarkable features of the economic literature is that demographic factors have so far entered in economic models almost exclusively through the size of population while the age composition of population has also important, and probably neglected, consequences for fluctuations in financial and macroeconomic variables. This paper has taken a step in the direction of linking fluctuations in the term structure of interest rates to the age structure of population.

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APPENDIX

We consider the following model specification for pricing bonds with macro and demographic factors:

$$
\begin{aligned}
y_{t,t+n} &= -\frac{1}{n}\left(A_n + B_n' X_t + \Gamma_n \mathrm{MY}_t^n\right) + \varepsilon_{t,t+1} \qquad \varepsilon_{t,t+n} \sim N(0,\sigma_n^2) \\
X_t &= \mu + \Phi X_{t-1} + \nu_t \qquad \nu_t \sim i.i.d.N(0,\Omega) \\
y_{t,t+1} &= \delta_0 + \delta_1' X_t + \delta_2 \mathrm{MY}_t \\
\Lambda_t &= \lambda_0 + \lambda_1 X_t \\
m_{t+1} &= \exp(-y_{t,t+1} - \frac{1}{2}\Lambda_t' \Omega \Lambda_t - \Lambda_t' v_{t+1}) \\
P_t^{(n)} &\equiv \left[\frac{1}{1+Y_{t,t+n}}\right]^n, \; y_{t,t+n} \equiv \ln\left(1+Y_{t,t+n}\right) \\
\Gamma_n \mathrm{MY}_t^n &\equiv \left[\gamma_0^n,\, \gamma_1^n \cdots, \gamma_{n-1}^n\right] \begin{bmatrix} \mathrm{MY}_t \\ \mathrm{MY}_{t+1} \\ \vdots \\ \mathrm{MY}_{t+n-1} \end{bmatrix} \quad X_t = \begin{bmatrix} f_t^\pi \\ f_t^x \\ f_t^{u,1} \\ f_t^{u,2} \\ f_t^{u,3} \end{bmatrix}
\end{aligned}
$$

Bond prices can be recursively computed as:

$$
\begin{aligned}
P_t^{(n)} &= E_t[m_{t+1} P_{t+1}^{(n-1)}] \\
&= E_t[m_{t+1} m_{t+2} P_{t+2}^{(n-2)}] \\
&= E_t[m_{t+1} m_{t+2} \cdots m_{t+n} P_{t+n}^{(0)}] \\
&= E_t[m_{t+1} m_{t+2} \cdots m_{t+n} 1] \\
&= E_t[\exp(\sum_{i=0}^{n-1}(-y_{t+i,t+i+1} - \frac{1}{2}\Lambda_{t+i}' \Omega \Lambda_{t+i} - \Lambda_{t+i}' \nu_{t+i+1}))] \\
&= E_t[\exp(A_n + B_n' X_t + \Gamma_n' \mathrm{MY}_t^n)] \\
&= E_t[\exp(-n y_{t,t+n})] \\
&= E_t^Q[\exp(-\sum_{i=0}^{n-1} y_{t+i,t+i+1})]
\end{aligned}
$$

where E_t^Q denotes the expectation under the risk-neutral probability measure, under which the dynamics of the state vector X_t are characterized by the risk neutral vector of constants μ^Q and by the autoregressive matrix Φ^Q

$$
\begin{aligned}
\mu^Q &= \mu - \Omega \lambda_0 \\
\Phi^Q &= \Phi - \Omega \lambda_1
\end{aligned}
$$

To derive the coefficients of the model, let us start with $n = 1$:

$$\begin{aligned} P_t^{(1)} &= \exp(-y_{t,t+1}) \\ &= \exp(-\delta_0 - \delta_1' X_t - \delta_2 \mathrm{MY}_t) \end{aligned}$$

$A_1 = -\delta_0$, $B_1 = -\delta_1$ and $\Gamma_1 = \gamma_0^1 = -\delta_2$,
Then for the general case $n + 1$,we have:

$$\begin{aligned} P_t^{(n+1)} &= E_t[m_{t+1} P_{t+1}^{(n)}] \\ &= E_t[\exp(-y_{t,t+1} - \frac{1}{2}\Lambda_t'\Omega\Lambda_t - \Lambda_t'\nu_{t+1})\exp(A_n + B_n' X_{t+1} + \Gamma_n \mathrm{MY}_{t+1}^n)] \\ &= \exp(-y_{t,t+1} - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n) E_t[\exp(-\Lambda_t'\nu_{t+1} + B_n' X_{t+1} + \Gamma_n \mathrm{MY}_{t+1}^n)] \\ &= \exp(-y_{t,t+1} - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + \Gamma_n \mathrm{MY}_{t+1}^n) \\ &\quad E_t[\exp(-\Lambda_t'\nu_{t+1} + B_n'(\mu + \Phi X_t + \nu_{t+1}))] \\ &= \exp[-\delta_0 - \delta_1' X_t - \delta_2 MY_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + \Gamma_n \mathrm{MY}_{t+1}^n + B_n'(\mu + \Phi X_t)] \\ &\quad E_t[\exp(-\Lambda_t'\nu_{t+1} + B_n'\nu_{t+1})] \\ &= \exp[-\delta_0 - \delta_1' X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n - \delta_2 \mathrm{MY}_t + B_n'(\mu + \Phi X_t) \\ &\quad + \Gamma_n \mathrm{MY}_{t+1}^n] \exp\{E_t[(-\Lambda_t' + B_n')\nu_{t+1}] + \frac{1}{2} var[(-\Lambda_t' + B_n')\nu_{t+1}]\} \\ &= \exp[-\delta_0 - \delta_1' X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + B_n'(\mu + \Phi X_t) \\ &\quad + \left[-\delta_2, \gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n\right] \mathrm{MY}_t^{n+1}] \exp\{\frac{1}{2} var[(-\Lambda_t' + B_n')\nu_{t+1}]\} \end{aligned}$$

To simplify the notation we define

$$[-\delta_2, \Gamma_n] \equiv \left[-\delta_2, \gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n\right]$$

$$
\begin{aligned}
&= \exp[-\delta_0 - \delta_1' X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + B_n'(\mu + \Phi X_t) \\
&+ [-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}] \exp\{\frac{1}{2}E_t[(-\Lambda_t' + B_n')\nu_{t+1}\nu_{t+1}^{'}(-\Lambda_t + B_n)]\} \\
&= \exp[-\delta_0 - \delta_1' X_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t + A_n + B_n'(\mu + \Phi X_t) \\
&+ [-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}] \exp\{\frac{1}{2}[\Lambda_t'\Omega\Lambda_t - 2B_n'\Omega\Lambda_t + B_n'\Omega B_n)]\} \\
&= \exp[-\delta_0 + A_n + B_n'\mu + (B_n'\Phi - \delta_1')X_t - B_n'\Omega\Lambda_t + \frac{1}{2}B_n'\Omega B_n \\
&+ [-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}] \\
&= \exp[-\delta_0 + A_n + B_n'\mu + (B_n'\Phi - \delta_1')X_t - B_n'\Omega(\lambda_0 + \lambda_1 X_t) + \frac{1}{2}B_n'\Omega B_n \\
&+ [-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}] \\
&= \exp[A_1 + A_n + B_n'(\mu - \Omega\lambda_0) + \frac{1}{2}B_n'\Omega B_n + (B_n'\Phi - B_n'\Omega\lambda_1 + B_1')X_t \\
&+ [-\delta_2, \Gamma_n]\mathrm{MY}_t^{n+1}]
\end{aligned}
$$

Then we can find the coefficients following the difference equations

$$
\begin{aligned}
A_{n+1} &= A_1 + A_n + B_n'(\mu - \Omega\lambda_0) + \frac{1}{2}B_n'\Omega B_n \\
B_{n+1}' &= B_n'\Phi - B_n'\Omega\lambda_1 + B_1' \\
\Gamma_{n+1} &= [-\delta_2, \Gamma_n]
\end{aligned}
$$

TABLE 1. Predictive regressions for the 1-year spot rate Monthly Data: 1958.06 − 2008.12

$$y_{t+12x,t+12x+12} \text{ -} y_{t,t+12} = a^x + b^x D_t + c^x [f_{t,t+12x,t+12x+12} \text{ -} y_{t,t+12}] + d^x [y_{t,t+12} \text{ -} P^i_{t,t+12}] + \varepsilon_{t+12x}$$

$$P^1_{t,t+12} = \frac{1}{60}\sum_{i=1}^{60} y_{t-i-1,t+12-i-1}\ (FAMA),\quad P^2_{t,t+12} = \frac{\sum_{i=1}^{120} 0.986^{i-1}\pi_{t-i-1}}{\sum_{i=1}^{120} 0.986^{i-1}} (CP),\ P^3_{t,t+12} = e^x \frac{1}{12}\sum_{i=1}^{12} MY_{t+i-1}$$

	a^x (s.e.)	b^x (s.e.)	c^x (s.e.)	d^x (s.e.)	e^x (s.e.)	R^2	
no cycle	−1.65 (0.16)	2.04 (0.20)	0.89 (0.09)			0.19	
Fama cycle no dummy	0.35 (0.15)		−0.47 (0.16)	−0.63 (0.09)		0.12	
Fama cycle	−0.93 (0.16)	2.43 (0.18)	−0.30 (0.14)	−0.83 (0.07)		0.33	$x = 2$
CP cycle	2.43 (0.22)		−0.86 (0.12)	−1.02 (0.07)		0.30	
MY cycle	13.66 (0.71)		−0.27 (0.08)	−0.92 (0.04)	−0.102 (0.005)	0.46	
no cycle	−2.75 (0.17)		3.27 (0.20)	1.54 (0.10)		0.38	
Fama cycle no dummy	−0.08 (0.17)		0.12 (0.418)	−0.49 (0.10)		0.13	
Fama cycle	−2.09 (0.17)	3.72 (0.19)	0.38 (0.14)	−0.835 (0.08)		0.48	$x = 3$
CP cycle	2.34 (0.25)		−0.57 (0.14)	−1.06 (0.08)		0.32	
MY cycle	16.72 (0.71)		−0.07 (0.08)	−1.13 (0.04)	−0.103 (0.005)	0.58	
no cycle	−3.44 (0.17)	4.13 (0.20)	1.83 (0.09)			0.49	
Fama cycle no dummy	−0.15 (0.19)		0.22 (0.20)	−0.48 (0.11)		0.14	
Fama cycle	−2.75 (0.16)	4.69 (0.18)	0.54 (0.13)	−0.95 (0.08)		0.60	$x = 4$
CP cycle	2.38 (0.28)		−0.52 (0.16)	−1.08 (0.09)		0.30	
MY cycle	18.90 (0.70)		−0.11 (0.08)	−1.29 (0.04)	−0.102 (0.005)	0.65	
no cycle	−3.46 (0.18)	4.38 (0.21)	1.66 (0.10)			0.47	
Fama cycle no dummy	0.09 (0.20)		−0.11 (0.21)	−0.56 (0.11)		0.10	
Fama cycle	−2.70 (0.17)	5.02 (0.19)	0.19 (0.14)	−1.07 (0.08)		0.61	$x = 5$
CP cycle	2.53 (0.30)		−0.70 (0.17)	−1.08 (0.09)		0.24	
MY	20.02 (0.71)		−0.40 (0.08)	−1.38 (0.04)	−0.098 (0.006)	0.66	

$f_{t,t+12x,t+12x+12}$:one-year forward rate at time t,with settlement in $x$years and maturity in $x+1$years, $y_{t,t+12}$ 1-year spot rate, π_t is annual core CPI inflation , MY_t is the ratio of middle aged to young population, D_t :dummy (1958:1-1981:8). Standard errors are Hansen-Hodrick(1980) adjusted.

TABLE 2. Encompassing regressions for the 1-year spot rate Monthly Data: 1958.06 – 2008.12

$$(y_{t+12x,t+12x+12} - y_{t,t+12}) = b^1 (y_{t+12x,t+12x+12} - y_{t,t+12})^{MY} + c^1 (y_{t+12x,t+12x+12} - y_{t,t+12})^{FAMA} + u_{t+12x}$$

$$(y_{t+12x,t+12x+12} - y_{t,t+12}) = b^2 (y_{t+12x,t+12x+12} - y_{t,t+12})^{MY} + c^2 (y_{t+12x,t+12x+12} - y_{t,t+12})^{CP} + v_{t+12x}$$

$$(y_{t+12x,t+12x+12} - y_{t,t+12})^{FAMA} = \hat{\beta}_0 + \hat{\beta}_1 y_{t,t+12} + \hat{\beta}_2 P^1_{t,t+12} + \hat{\beta}_3 D_t$$

$$(y_{t+12x,t+12x+12} - y_{t,t+12})^{CP} = \hat{\gamma}_0 + \hat{\gamma}_1 y_{t,t+12} + \hat{\gamma}_2 P^2_{t,t+12}$$

$$(y_{t+12x,t+12x+12} - y_{t,t+12})^{MY} = \hat{\delta}_0 + \hat{\delta}_1 y_{t,t+12} + \hat{\delta}_2 P^3_{t,t+12}$$

$$P^1_{t,t+12} = \frac{1}{60}\sum_{i=1}^{60} y_{t-i-1,t+12-i-1}\ (FAMA), \quad P^2_{t,t+12} = \frac{\sum_{i=1}^{120} 0.986^{i-1}\pi_{t-i-1}}{\sum_{i=1}^{120} 0.986^{i-1}} (CP), \quad P^3_{t,t+12} = e^x \frac{1}{12}\sum_{i=1}^{12} MY_{t+i-1}$$

b^1 (s.e.)	c^1 (s.e.)	b^2 (s.e.)	c^2 (s.e.)	
1.07 (0.13)	−0.09 (0.11)	0.88 (0.059)	0.25 (0.078)	$x = 2$
0.91 (0.09)	0.09 (0.10)	0.93 (0.05)	0.11 (0.06)	$x = 3$
0.67 (0.09)	0.35 (0.09)	0.99 (0.06)	0.02 (0.04)	$x = 4$
0.53 (0.08)	0.50 (0.08)	1.01 (0.04)	−0.03 (0.07)	$x = 5$

Standard errors are Hansen-Hodrick(1980) adjusted.

TABLE 3. Summary Statistics of the Data in Term Structure Models

	Central Moments				Autocorrelations		
	mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
3-month	5.850	2.778	1.061	4.775	0.917	0.866	0.843
1-year	6.282	2.748	0.817	4.107	0.925	0.881	0.841
2-year	6.483	2.698	0.853	3.980	0.935	0.891	0.852
3-year	6.643	2.615	0.876	3.915	0.941	0.899	0.860
4-year	6.774	2.571	0.928	3.950	0.945	0.904	0.865
5-year	6.851	2.521	0.910	3.680	0.949	0.910	0.875
inflation factor	0.116	0.922	2.149	16.463	0.337	0.100	0.170
real factor	−0.026	0.965	1.225	6.631	0.681	0.450	0.248
middle-young	0.836	0.211	0.102	1.500	0.995	0.989	0.981

1, 4, 8, 12, 16, 20 quarter yields are annualized (in percentage) zero coupon bond yields from the Fama-Bliss CRSP bond files. Inflation and real activity refer to the price and output factors extracted from large dataset provided by Ludvigson and Ng (2009). Sample 1964Q1-2007Q4.

TABLE 4. Model Estimates. Sample 1964Q1-2007Q4.

Panel A. Model without Demographics

	B_0 (s.e.)	B_1^y (s.e.)	B_1^π (s.e.)	B_1^x (s.e.)	
π_t	−0.152 (−0.875)	0.032 (0.448)	0.304 (0.080)	0.147 (0.103)	
x_t	−0.270 (0.136)	0.216 (0.056)	−0.085 (0.063)	0.725 (0.081)	

$y_{t,t+1} = (1-\varphi)\left(\rho_0 + \rho_2\pi_{t-1} + \rho_3 x_{t-1}\right) + \varphi y_{t-1,t} + u_{1t}$

	ρ_0 (s.e.)	ρ_1	ρ_2 (s.e.)	ρ_3 (s.e.)	φ (s.e.)
$y_{t,t+1}$	5.429 (1.11)	0	2.440 (1.45)	−3.034 (1.84)	0.926 (0.03)

	1.89	0.05	−0.22
$cov\left(u_{1t}, u_{2t}, u_{2t}\right)$		0.41	−0.19
			0.57

$y_{t,t+n} = y^*_{t,t+n} + k_n + u_{4t} \quad y^*_{t,t+n} = \frac{1}{n}\sum_{i=0}^{n-1} E_t[y_{t+i,t+i+1} I_t]$

	k_1 (s.e.)	k_2 (s.e.)	k_3 (s.e.)	k_4 (s.e.)	k_5 (s.e.)
$y_{t,t+n}$	0.4371 (0.062)	0.6549 (0.098)	0.8503 (0.125)	1.0300 (0.143)	1.1504 (0.152)

Panel B. Model with Demographics

	B_0 (s.e.)	B_1^y (s.e.)	B_1^π (s.e.)	B_1^x (s.e.)	
π_t	−0.152 (−0.875)	0.032 (0.448)	0.304 (0.080)	0.147 (0.103)	
x_t	−0.270 (0.136)	0.216 (0.056)	−0.085 (0.063)	0.725 (0.081)	

$y_{t,t+1} = (1-\varphi)\left(\rho_0 + \rho_1 MY_t + \rho_2\pi_{t-1} + \rho_3 x_{t-1}\right) + \varphi y_{t-1,t} + u_{1t}$

	ρ_0 (s.e.)	ρ_1 (s.e.)	ρ_2 (s.e.)	ρ_3 (s.e.)	φ (s.e.)
$y_{t,t+1}$	13.262 (2.228)	−0.091 (0.025)	1.225 (0.664)	−1.276 (0.757)	0.853 (0.041)

	1.06	−0.03	−0.17
$cov\left(u_{1t}, u_{2t}, u_{2t}\right)$		0.67	−0.05
			0.43

$y_{t,t+n} = y^*_{t,t+n} + k_n + u_{4t} \quad y^*_{t,t+n} = \frac{1}{n}\sum_{i=0}^{n-1} E_t[y_{t+i,t+i+1} I_t]$

	k_1 (s.e.)	k_2 (s.e.)	k_3 (s.e.)	k_4 (s.e.)	k_5 (s.e.)
$y_{t,t+n}$	0.4326 (0.061)	0.6470 (0.097)	0.8393 (0.124)	1.0167 (0.143)	1.1357 (0.153)

This table reports the least squares estimation results for system (7) in the paper. Panel A reports the estimation results of a model without demographics, while panel B is the results for model with demographics. Standard errors are reported within brackets.

TABLE 5. Yield Forecasts Comparison					
Panel A. FRMSE (Random-walk)					
horz	4	8	12	16	20
$y_{t,t+1}$	**0.8221** (0.064)	**0.6667** (0.000)	**0.6407** (0.000)	**0.6584** (0.000)	**0.7099** (0.001)
$y_{t,t+4}$	**0.9656** (0.033)	**0.6602** (0.050)	**0.5349** (0.028)	**0.4925** (0.012)	**0.4853** (0.010)
$y_{t,t+8}$	**0.9866** (0.002)	**0.6603** (0.008)	**0.5306** (0.011)	**0.4847** (0.021)	**0.4741** (0.043)
$y_{t,t+12}$	**1.0189** (0.000)	**0.6699** (0.000)	**0.5340** (0.001)	**0.4883** (0.002)	**0.4699** (0.004)
$y_{t,t+16}$	**1.0573** (0.000)	**0.6857** (0.001)	**0.5430** (0.004)	**0.4883** (0.006)	**0.4713** (0.009)
$y_{t,t+20}$	**1.0933** (0.002)	**0.7036** (0.014)	**0.5554** (0.022)	**0.4976** (0.023)	**0.4788** (0.022)
Panel B. FRMSE (Without demographics)					
horz	4	8	12	16	20
$y_{t,t+1}$	**0.7819** (0.000)	**0.7514** (0.001)	**0.7534** (0.008)	**0.7671** (0.010)	**0.7878** (0.001)
$y_{t,t+4}$	**0.7380** (0.019)	**0.6820** (0.037)	**0.6490** (0.058)	**0.6361** (0.094)	**0.6228** (0.137)
$y_{t,t+8}$	**0.6977** (0.014)	**0.6484** (0.033)	**0.6171** (0.046)	**0.6041** (0.072)	**0.5901** (0.108)
$y_{t,t+12}$	**0.6727** (0.007)	**0.6268** (0.027)	**0.5961** (0.043)	**0.5823** (0.066)	**0.5672** (0.098)
$y_{t,t+16}$	**0.6577** (0.003)	**0.6139** (0.018)	**0.5833** (0.034)	**0.5685** (0.054)	**0.5522** (0.083)
$y_{t,t+20}$	**0.6499** (0.001)	**0.6088** (0.008)	**0.5793** (0.018)	**0.5647** (0.032)	**0.5482** (0.054)

We use the in-sample estimators, from 1964Q4 to 1982Q4, to generate out-of sample forecasts until 2007Q4. h indicates 4, 8, 12, 16, 20 quarter out-of-sample forecasts. We measure forecasting performance as the ratio of the root mean squared forecast error (RMSFE) of our model to the RMSFE of a random walk forecast and the benchmark model without demographics. The table shows better forecasts with respect to the random walk and yield-macro model with bold characters for the range of [0.9, 1,) and with added underline for ratios smaller than 0.9. We report in parentheses the p-values of the forecasting test due to Giacomini and White (2006). A p-value below 0.01 (0.05, 0.10) indicates a significant difference in forecasting performance at the 1% (5%, 10%) level.

TABLE 6. Out-of Sample R^2 Comparison

Panel A. Model without Demographics

horz	4	8	12	16	20
$y_{t,t+1}$	−0.1054	0.2127	0.2769	0.2634	0.1880
$y_{t,t+4}$	−0.7123	0.0631	0.3209	0.4005	0.3927
$y_{t,t+8}$	−0.9996	−0.0370	0.2606	0.3563	0.3545
$y_{t,t+12}$	−1.2941	−0.1422	0.1973	0.3096	0.3135
$y_{t,t+16}$	−1.5847	−0.2478	0.1335	0.2621	0.2713
$y_{t,t+20}$	−1.8299	−0.3357	0.0809	0.2234	0.2371

Panel B. Model with Demographics

horz	4	8	12	16	20
$y_{t,t+1}$	0.3242	0.5555	0.5895	0.5666	0.4961
$y_{t,t+4}$	0.0675	0.5642	0.7139	0.7574	0.7645
$y_{t,t+8}$	0.0267	0.5641	0.7185	0.7651	0.7752
$y_{t,t+12}$	−0.0382	0.5512	0.7148	0.7659	0.7792
$y_{t,t+16}$	−0.1179	0.5298	0.7052	0.7615	0.7778
$y_{t,t+20}$	−0.1953	0.5050	0.6915	0.7523	0.7708

We use the in-sample estimation, from 1964Q4 to 1982Q4, for models with and without demographics to generate out-of sample forecasts until 2007Q4. h indicates 4, 8, 12, 16, 20 quarter out-of-sample forecasts. We measure forecasting performance via the out-of-sample R^2_{OS} (Campbell and Thomson, 2008)

TABLE 7. Model Estimates (time-varying risk price). Sample 1969Q1-2007Q4.										
	Unrestricted Model					Restricted Model				
Companion form Φ										
	0.222 (0.090)	−0.118 (0.080)	−0.043 (0.050)	0.007 (0.092)	−0.038 (0.157)	0.213 (0.937)	−0.095 (1.796)	−0.044 (0.072)	0.011 (1.032)	−0.049 (0.533)
	0.207 (0.086)	0.521 (0.111)	−0.102 (0.115)	0.021 (0.063)	−0.054 (0.364)	0.179 (1.850)	0.583 (0.327)	−0.083 (0.572)	0.039 (1.348)	−0.105 (0.188)
	0.223 (0.115)	0.500 (0.129)	0.907 (0.239)	−0.083 (0.335)	0.063 (0.864)	−0.010 (2.285)	−0.047 (1.340)	0.962 (0.175)	0.002 (1.264)	0.027 (0.579)
	0.186 (0.363)	0.114 (0.374)	−0.103 (0.435)	0.766 (1.229)	−0.153 (0.699)	−0.212 (0.358)	−0.018 (4.459)	0.010 (1.490)	0.554 (0.232)	0.258 (1.372)
	0.144 (0.602)	0.116 (0.446)	−0.091 (0.402)	−0.234 (0.476)	0.679 (1.243)	0.316 (0.247)	0.482 (2.200)	0.137 (0.392)	0.081 (2.002)	0.805 (0.418)
Short rate parameters										
δ_1	−0.240 (0.109)	−0.542 (0.602)	2.565 (1.057)	0.000 (0.000)	0.901 (2.562)	−0.255 (2.473)	−0.569 (1.323)	0.981 (1.948)	0.000 (0.000)	2.520 (1.129)
δ_2	−0.022 (0.017)					0				
Price of risk λ_0 and λ_1										
$(\lambda_0)^T$	−0.000 (0.024)	−0.003 (0.048)	−0.002 (0.019)	0.001 (0.011)	−0.000 (0.001)	0.004 (0.555)	−0.003 (0.375)	0.000 (0.110)	−0.000 (0.092)	−0.001 (0.034)
λ_1	−0.184 (0.057)		$\cdots$		**0**	−0.158 (1.038)		$\cdots$		**0**
		0.000 (0.002)					0.013 (0.090)			
	$\vdots$		−0.141 (0.134)		$\vdots$	$\vdots$		−0.018 (0.567)		$\vdots$
				−0.064 (0.200)					−0.046 (0.329)	
	0		$\cdots$		0.059 (0.457)	**0**		$\cdots$		−0.137 (0.328)
Under risk-neutral $\Phi^Q = \Phi - \Omega\lambda_1$										
	1.051	0.207	0.223	0.186	0.144	0.923	0.183	−0.010	−0.212	0.316
	−0.181	0.520	0.500	0.114	0.116	−0.145	0.545	−0.047	−0.018	0.482
	−0.043	−0.102	1.048	−0.103	−0.091	−0.044	−0.083	0.979	0.010	0.137
	0.007	0.021	−0.083	0.830	−0.234	0.011	0.039	0.002	0.600	0.082
	−0.038	−0.054	0.063	−0.153	0.620	−0.049	−0.105	0.028	0.258	0.943
Innovation covariance matrix Ω^o										
	4.502	−0.397				4.511	−0.318			
		2.674					2.840			

This table reports the parameter estimates of maximum likelihood estimation of the no-arbitrage models, including macro factors and middle-aged-young ratio based on restricted model and unrestricted model, respectively. Standard errors within brackets

TABLE 8. Yield Forecasts						
Panel A. FRMSE (Random-walk)						
horz	1	4	8	12	16	20
$y_{t,t+1}$	1.3641 (0.214)	**0.9951** (0.000)	**0.8620** (0.000)	**0.7711** (0.000)	**0.7476** (0.000)	**0.8027** (0.000)
$y_{t,t+4}$	1.3681 (0.000)	1.0758 (0.032)	**0.8936** (0.000)	**0.8020** (0.006)	**0.7863** (0.001)	**0.8669** (0.001)
$y_{t,t+8}$	1.2335 (0.000)	1.0737 (0.154)	**0.9251** (0.004)	**0.8550** (0.011)	**0.8543** (0.010)	**0.9646** (0.000)
$y_{t,t+12}$	1.1409 (0.000)	1.0499 (0.049)	**0.9452** (0.441)	**0.8912** (0.057)	**0.9005** (0.010)	1.0264 (0.039)
$y_{t,t+16}$	1.1290 (0.000)	1.0385 (0.015)	**0.9710** (0.184)	**0.9376** (0.430)	**0.9487** (0.000)	1.0805 (0.023)
$y_{t,t+20}$	1.1251 (0.000)	1.0251 (0.005)	**0.9935** (0.063)	**0.9725** (0.106)	**0.9836** (0.229)	1.1151 (0.02)
Panel B. FRMSE (Restricted Model)						
$y_{t,t+1}$	1.0909 (0.003)	**0.8574** (0.046)	**0.8261** (0.079)	**0.8280** (0.076)	**0.8017** (0.102)	**0.7443** (0.056)
$y_{t,t+4}$	1.0193 (0.001)	**0.8674** (0.027)	**0.8428** (0.047)	**0.8411** (0.064)	**0.8090** (0.054)	**0.7568** (0.024)
$y_{t,t+8}$	1.0022 (0.000)	**0.8677** (0.005)	**0.8442** (0.007)	**0.8442** (0.009)	**0.8094** (0.008)	**0.7614** (0.011)
$y_{t,t+12}$	1.0075 (0.000)	**0.8683** (0.000)	**0.8413** (0.000)	**0.8401** (0.000)	**0.8028** (0.000)	**0.7571** (0.000)
$y_{t,t+16}$	1.0099 (0.000)	**0.8645** (0.000)	**0.8342** (0.001)	**0.8304** (0.001)	**0.7904** (0.000)	**0.7465** (0.000)
$y_{t,t+20}$	1.0069 (0.000)	**0.8594** (0.000)	**0.8225** (0.006)	**0.8180** (0.010)	**0.7740** (0.000)	**0.7324** (0.000)
Panel C. FRMSE (Unit Root Level)						
$y_{t,t+1}$	1.2164 (0.0329)	1.0083 (0.0418)	**0.8870** (0.0185)	**0.8896** (0.0137)	**0.9134** (0.0176)	**0.9584** (0.0177)
$y_{t,t+4}$	1.2476 (0.0003)	1.0172 (0.0015)	**0.9239** (0.009)	**0.9220** (0.007)	**0.9331** (0.000)	**0.9600** (0.000)
$y_{t,t+8}$	1.1929 (0.006)	**0.9966** (0.0101)	**0.9441** (0.0137)	**0.9434** (0.0164)	**0.9486** (0.0530)	**0.9631** (0.0861)
$y_{t,t+12}$	1.1572 (0.000)	**0.9796** (0.0024)	**0.9518** (0.0046)	**0.9452** (0.0090)	**0.9428** (0.0201)	**0.9460** (0.0542)
$y_{t,t+16}$	1.1361 (0.000)	**0.9636** (0.002)	**0.9567** (0.000)	**0.9465** (0.000)	**0.9330** (0.000)	**0.9257** (0.000)
$y_{t,t+20}$	1.1080 (0.000)	**0.9380** (0.0011)	**0.9451** (0.0017)	**0.9330** (0.006)	**0.9075** (0.000)	**0.8922** (0.000)

h indicates 1, 4, 8, 12, 16, 20 quarter out-of-sample forecasts. We measure forecasting performance as the ratio of the root mean squared forecast error (RMSFE) of our model to the RMSFE of a random walk forecast and the benchmark yield-macro model. bold characters highlight a ratio in the range of [0.9, 1,) and bold and underlined characters highlight a ratio smaller than 0.9. We report in parentheses the p-values of the forecasting test due to Giacomini and White (2006). A p-value below 0.01 (0.05, 0.10) indicates a significant difference in forecasting performance at the 1% (5%, 10%) level.

TABLE 9. Out-of Sample R^2 Comparison						
Panel A. Restricted Model						
horz	1	4	8	12	16	20
$y_{t,t+1}$	−0.5637	−0.3470	−0.0889	0.1327	0.1304	−0.1630
$y_{t,t+4}$	−0.8015	−0.5384	−0.1241	0.0908	0.0553	−0.3122
$y_{t,t+8}$	−0.5148	−0.5309	−0.2007	−0.0258	−0.1139	−0.6050
$y_{t,t+12}$	−0.2823	−0.4620	−0.2620	−0.1254	−0.2580	−0.8379
$y_{t,t+16}$	−0.2498	−0.4431	−0.3550	−0.2748	−0.4409	−1.0949
$y_{t,t+20}$	−0.2487	−0.4230	−0.4590	−0.4135	−0.6147	−1.3184
Panel B. Unrestricted Model						
$y_{t,t+1}$	−0.8609	0.0098	0.2569	0.4054	0.4411	0.3557
$y_{t,t+4}$	−0.8716	−0.1573	0.2014	0.3568	0.3817	0.2484
$y_{t,t+8}$	−0.5214	−0.1527	0.1443	0.2689	0.2703	0.0695
$y_{t,t+12}$	−0.3016	−0.1023	0.1067	0.2057	0.1892	−0.0534
$y_{t,t+16}$	−0.2746	−0.0785	0.0571	0.1209	0.0999	−0.1674
$y_{t,t+20}$	−0.2660	−0.0509	0.0130	0.0542	0.0326	−0.2435
Panel C. Model with Unit Root Level Factor						
$y_{t,t+1}$	−0.2577	0.0261	0.0555	0.2488	0.3300	0.2985
$y_{t,t+4}$	−0.2024	−0.1185	0.0646	0.2434	0.2899	0.1845
$y_{t,t+8}$	−0.0692	−0.1607	0.0399	0.1786	0.1891	−0.0031
$y_{t,t+12}$	0.0280	−0.1487	0.0140	0.1109	0.0878	−0.1771
$y_{t,t+16}$	0.0124	−0.1616	−0.0302	0.0188	−0.0341	−0.3624
$y_{t,t+20}$	−0.0312	−0.1945	−0.1049	−0.0864	−0.1746	−0.5622

We use the in-sample estimation for each models with or without demographics, from 1964Q4 to 1982Q4, to generate out-of sample forecasts until 2007Q4. h indicates 4, 8, 12, 16, 20 quarter out-of-sample forecasts. We measure forecasting performance as the out-of-sample R^2_{OS} (Campbell and Thomson, 2008)

TABLE 10. Model Estimates. Sample 1964Q1-2007Q4.					
Panel A. Demographics based permanent component					
$\pi_t = \beta_1 \pi_{t-1} + (1-\beta_1)\,\pi_t^* + \varepsilon_{1t}$					
$\pi_t^* = \alpha_0 + \alpha_1 MY_t$					
β_1 (s.e.)	α_0 (s.e.)	α_1 (s.e.)	R^2	$\sigma(\varepsilon_{1t})$	$\mu(\pi_t)$
0.931 (0.028)	11.94 (2.89)	−0.089 (0.033)	0.94	0.65	4.42
Panel B. Cieslak-Povala's permanent component					
$\pi_t = \beta_0 + \beta_1 \pi_{t-1} + (1-\beta_1)\,\pi_t^* + \varepsilon_{2t}$					
$\pi_t^* = \pi_{t-1}^* + (1-0.96)\left(\pi_t - \pi_{t-1}^*\right)$					
β_1 (s.e.)	β_0 (s.e.)		R^2	$\sigma(\varepsilon_{2t})$	$\mu(\pi_t)$
0.992 (0.023)	0.31 (0.134)		0.94	0.65	4.42
Panel C. Short rate equation with demographics					
$y_{t,t+1} = \rho_1 y_{t-1,t} + (1-\rho_1)\,y_{t,t+1}^* + v_t$					
$y_{t,t+1}^* = \rho_0 + \rho_2 MY_t$					
ρ_1 (s.e.)	ρ_0 (s.e.)	ρ_2 (s.e.)	R^2	$\sigma(v_t)$	$\mu(y_{t,t+1})$
0.842 (0.040)	13.857 (2.084)	−0.096 (0.024)	0.85	1.06	5.86

Panel A reports the estimation results of a model with a demographics-based permanent component, while panel B contains the results for a model with Cieslak-Povala's permanent component. In Panel C, we report the results of the estimation of an autoregressive model for the short rate, augmented with MY. Standard errors in parentheses. Sample 1964Q1-2007Q4.

TABLE 11. The Excess Returns and Demographic Variable

$$xr^n_{t,t+1} = b_0 + b^1_1 y_{t,t+1} + \cdots + b^5_1 y_{t,t+5} + b_2 MY_t + \varepsilon^n_{t,t+1}$$

Sample 1952:6-2008:12

	b_0 (t−stat)	b^1_1 (t−stat)	b^2_1 (t−stat)	b^3_1 (t−stat)	b^4_1 (t−stat)	b^5_1 (t−stat)	b_2 (t−stat)	R^2
Panel A. Unrestricted Model								
$n=2$	−12.07	−0.54	−0.50	4.60	−3.06	0.29	8.92	0.46
	(−4.13)	(−0.57)	(−0.51)	(2.06)	(−1.87)	(0.23)	(3.40)	
$n=3$	−21.85	−1.32	−1.49	9.14	−5.57	0.60	16.34	0.44
	(−4.17)	(−0.73)	(−0.79)	(2.34)	(−1.79)	(0.24)	(3.44)	
$n=4$	−30.94	−2.03	−2.69	12.18	−6.57	1.01	22.70	0.45
	(−4.42)	(−0.89)	(−1.17)	(2.49)	(−1.64)	(0.31)	(3.58)	
$n=5$	−38.31	−3.26	−1.87	13.26	−8.92	3.08	27.88	0.44
	(−4.54)	(−1.09)	(−0.51)	(2.31)	(−1.85)	(0.75)	(3.65)	
Panel B. Restricted Model I ($b_2 = 0$)								
$n=2$	−1.94	−1.35	−0.16	4.31	−0.96	−1.48	0	0.20
	(−2.40)	(−1.23)	(−0.11)	(1.55)	(−0.43)	(−0.86)		
$n=3$	−3.28	−2.79	−0.89	8.61	−1.72	−2.64	0	0.17
	(−2.24)	(−1.36)	(−0.32)	(1.76)	(−0.40)	(−0.81)		
$n=4$	−5.14	−4.09	−1.85	11.45	−1.22	−3.49	0	0.18
	(−2.55)	(−1.52)	(−0.52)	(1.83)	(−0.22)	(−0.80)		
$n=5$	−6.63	−5.78	−0.83	12.35	−2.35	−2.44	0	0.18
	(−2.66)	(−1.63)	(−0.17)	(1.74)	(−0.35)	(−0.44)		
Panel C. Restricted Model II								
$n=2$	−1.23	0	0	0	0	0	1.76	0.02
	(−0.43)						(0.61)	
$n=3$	−3.35	0	0	0	0	0	4.48	0.04
	(−0.63)						(0.83)	
$n=4$	−5.22	0	0	0	0	0	6.74	0.04
	(−0.70)						(0.90)	
$n=5$	−7.30	0	0	0	0	0	9.04	0.05
	(−0.77)						(0.96)	

Panel A reports the least square estimation results of a model with demographics, while panel B is the least square estimation results for model without demographics. Hansen-Hodrick t-statistic are reported within brackets. Sample 1952:6-2008:12, monthly data.

TABLE 12. Pooled Panel Regressions

Benchmark model: $x_t = \alpha_0 + \alpha_1 x_{t-1} + \varepsilon_t$

Augmented model: $x_t = \beta_0 + \beta_1 x_{t-1} + \beta_2 \mathrm{MY}_t + \varepsilon_t$

Specification	π_{t-1}	R_{lt-1}	MY_t	$\bar{\mathrm{R}}^2$
Panel A. Annual Inflation, π_t,n=39				
(1)	0.742 (18.56)			0.65
(2)	0.691 (19.70)		−0.029 (−4.58)	0.67
Panel B. Nominal Bond Yield, R_{lt},n=36				
(1)		0.907 (29.86)		0.87
(2)		0.830 (29.12)	−0.021 (−6.28)	0.87

Pooled GLS regression coefficients with country fixed effects (Cross-section weights and White heteroscedasticity adjusted t-statistics in parentheses). For each series, annual inflation π_t, long term nominal bond yield R_{lt}. Specification (1) is the benchmark model and specification (2) is the augmented model with MY_t. Annual sample 1961-2009 (Unbalanced panel).

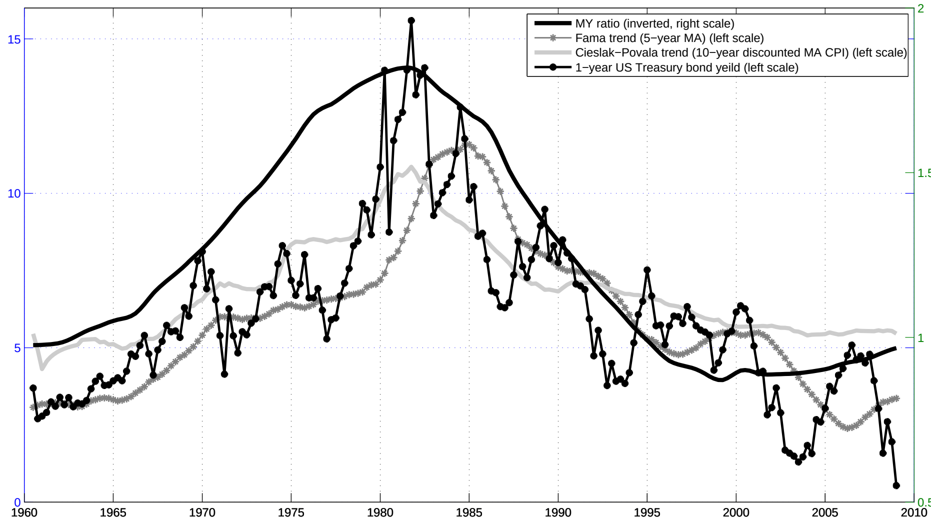


Figure 1: 1-Year US Treasury bond yields and their permament component.

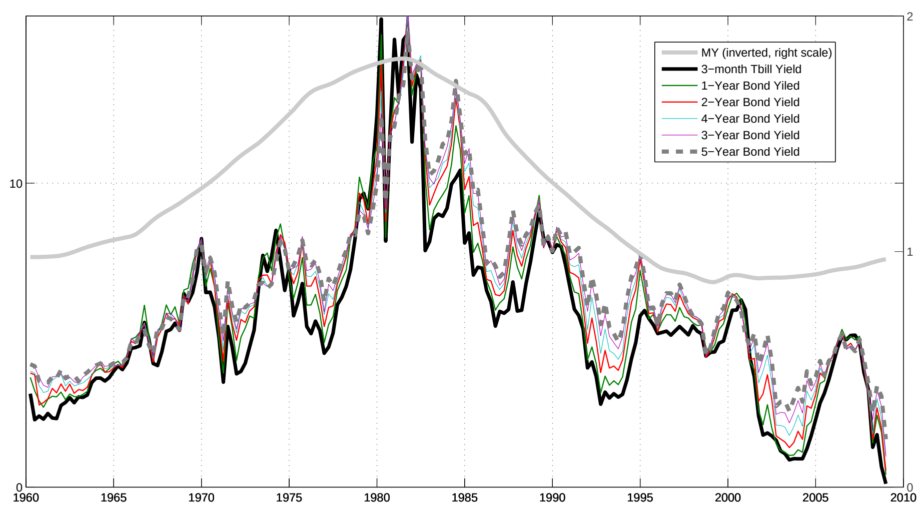


Figure 2. US post-war nominal interest rates and inverted MY

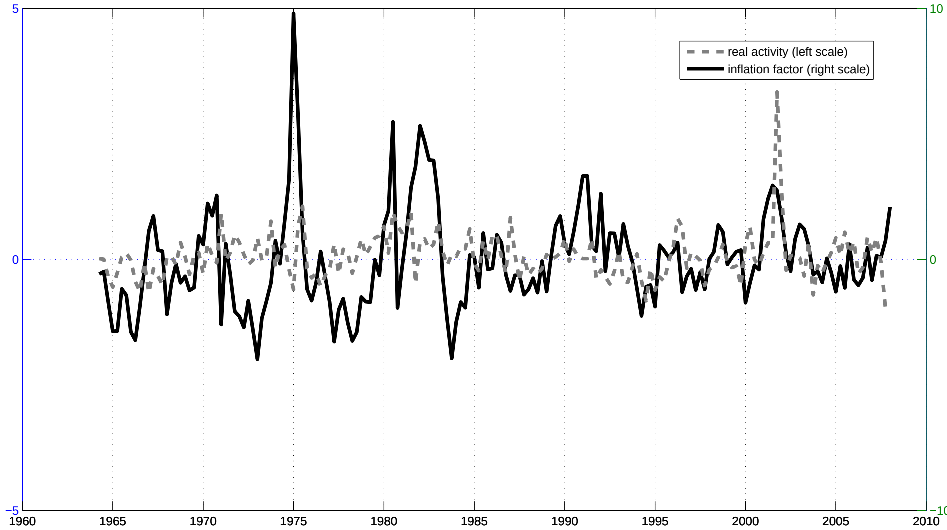


Figure 3. Nominal and Real Macroeconomic factors. Real activity and inflation factors extracted from a large dataset (Ludvigson and Ng (2009)).

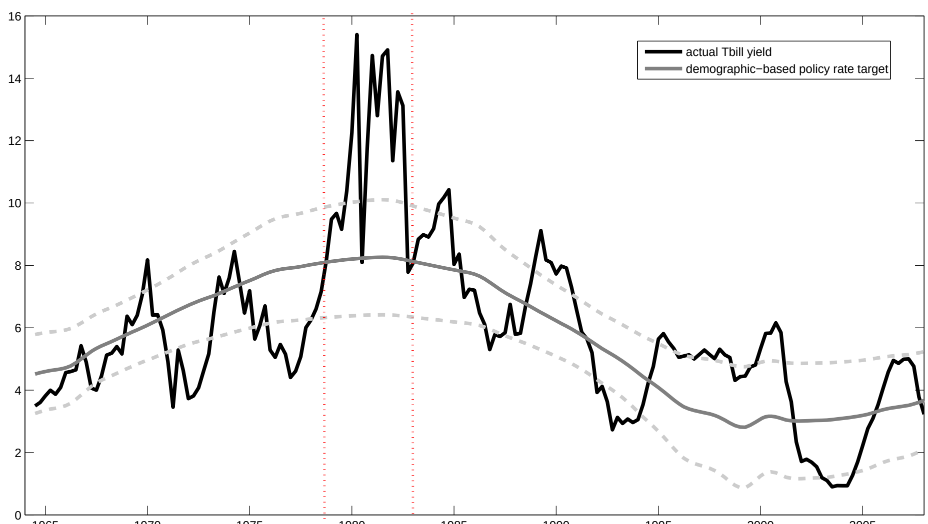
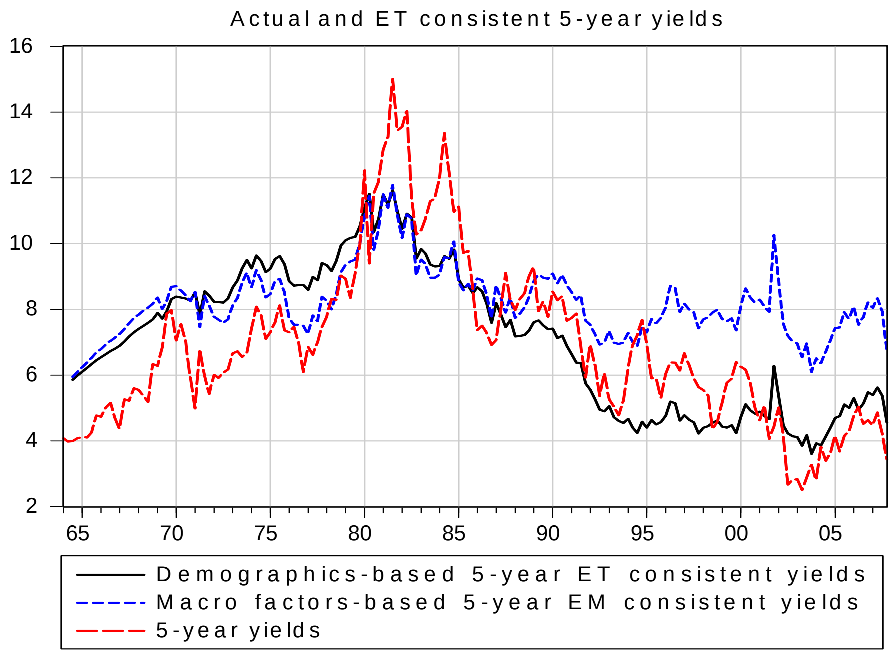


Figure 4. Actual 3-month TBill yield and estimated demographic-based policy rate target with its 95 per cent confidence interval.



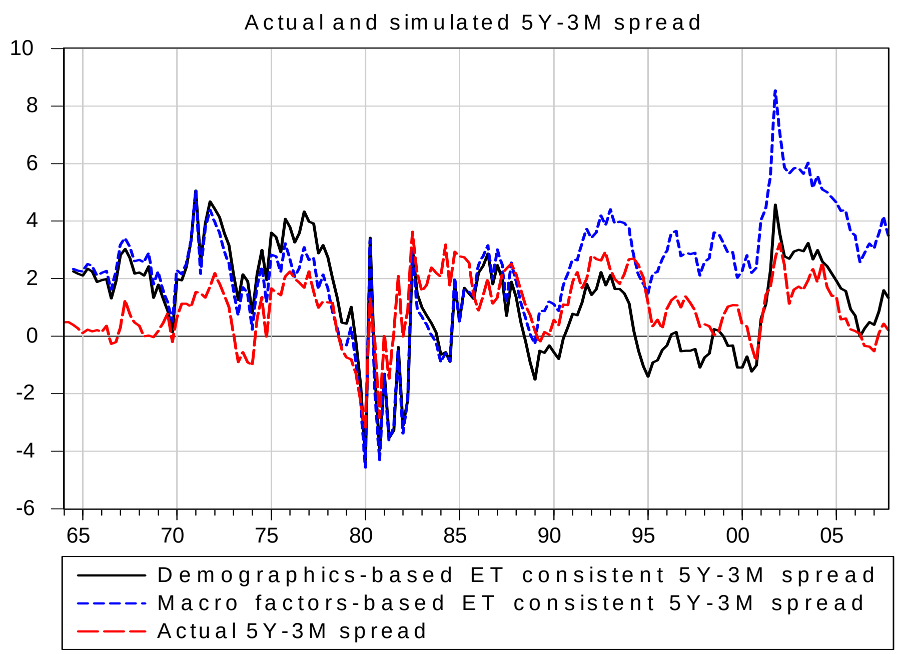


Figure 5. Actual and 5-year ET consistent yields and actual and simulated 5Y-3M spread.

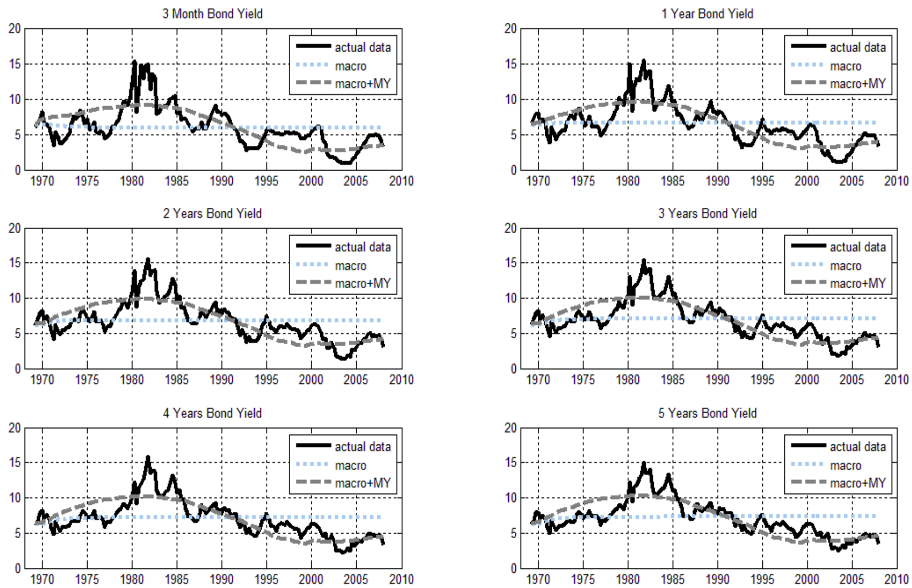


Figure 6. Dynamic Simulations. This figure plots the time series for actual bond yields (maturity: 3m, 1y, 2y, 3y, 4y, 5y) alongwith those dynamically simulated from the benchmark model with macro factors (dotted light blue line) and that augmented with demographics (dashed grey line). The models are are estimated over the full sample and dynamically solved from the first observation onward.

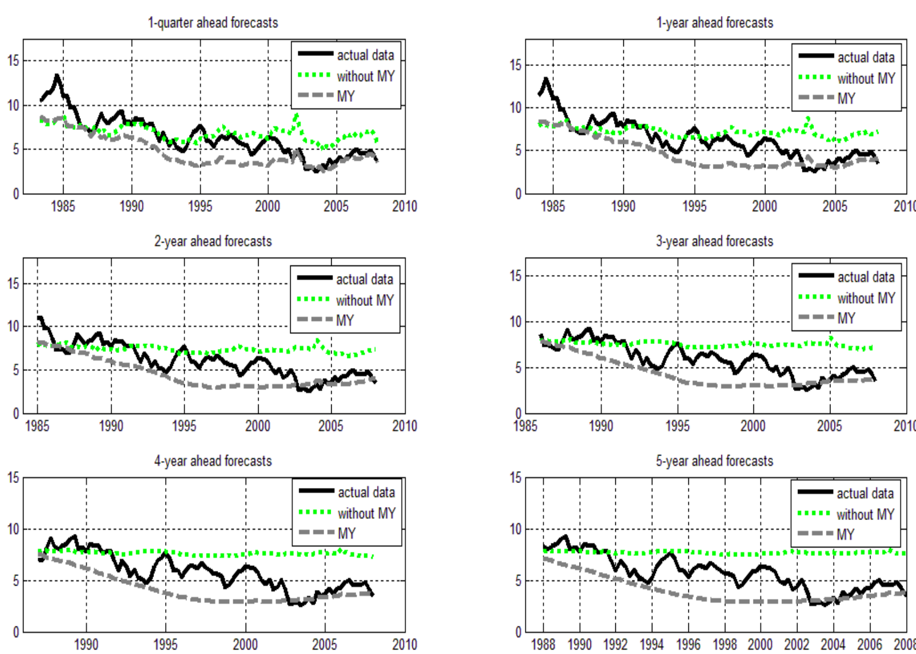


Figure 7. Out-of Sample Forecasting of 5-year bond yields. out-of sample forecasted series generated by the weak Expectations Theory model without demographics and with demographics.

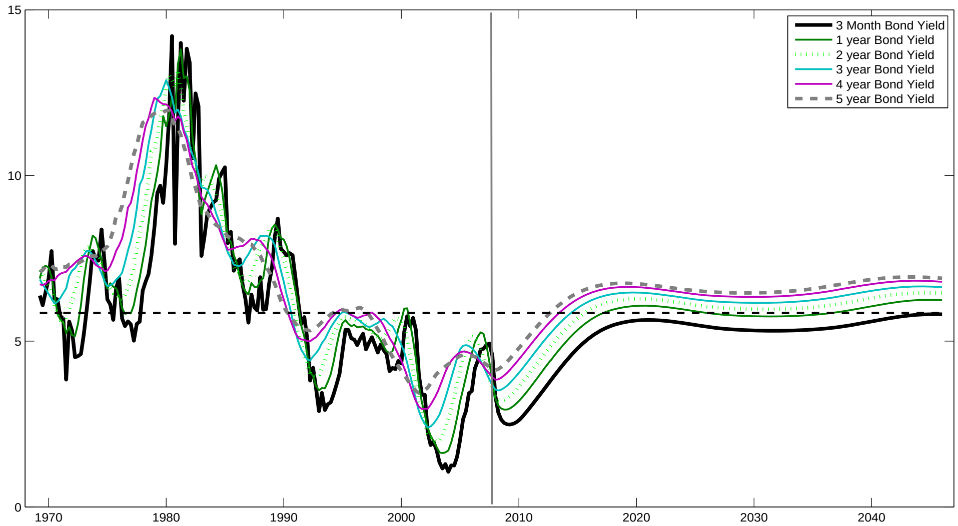


Figure 8. In Sample fitted values (1969Q1-2007Q4)and out of sample dynamically simulated values (2008Q1-2045Q4) for yield at all maturities based on the expectation theory model with demographics.

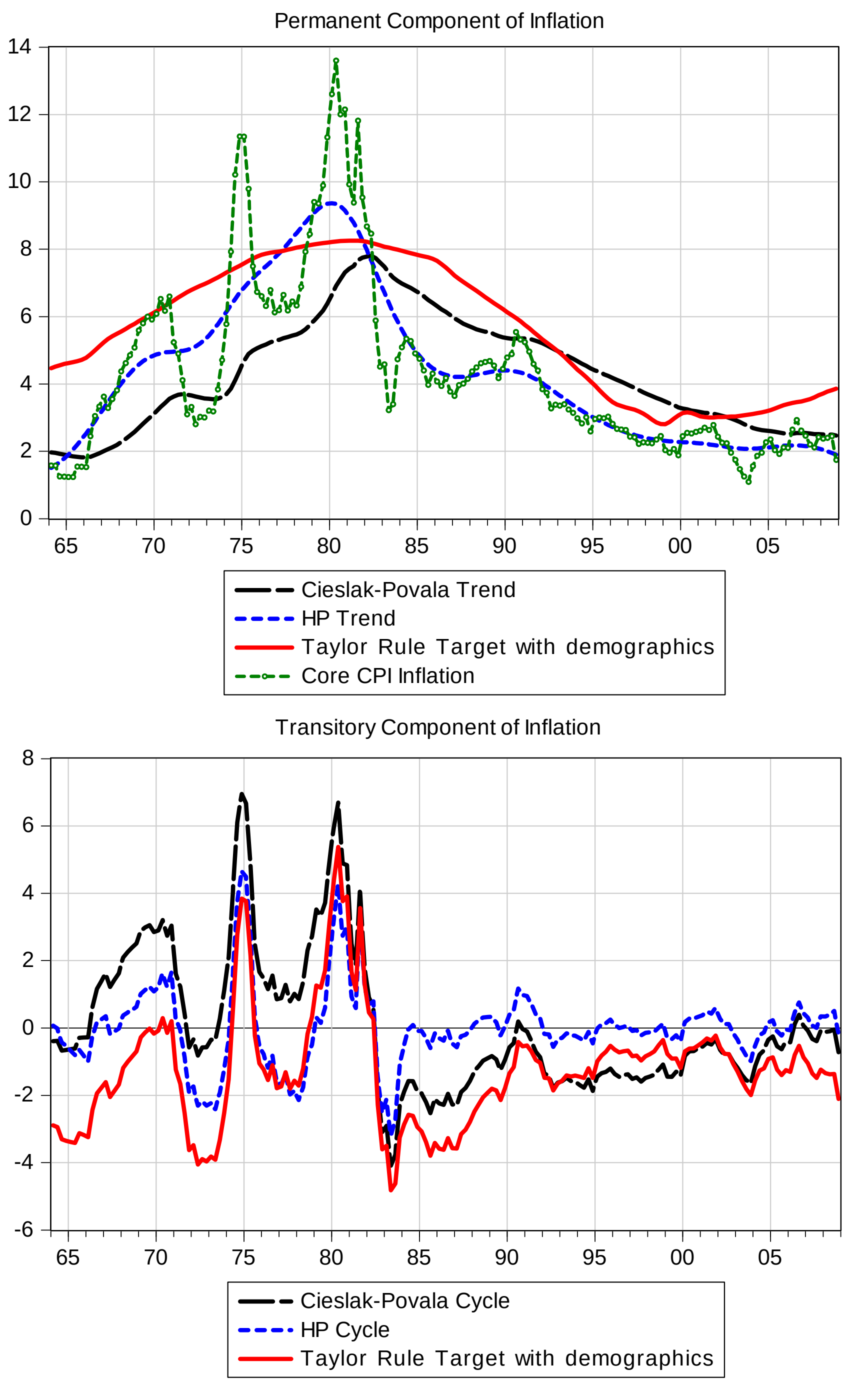


Figure 9. Permanent and transitory components of annual core inflation

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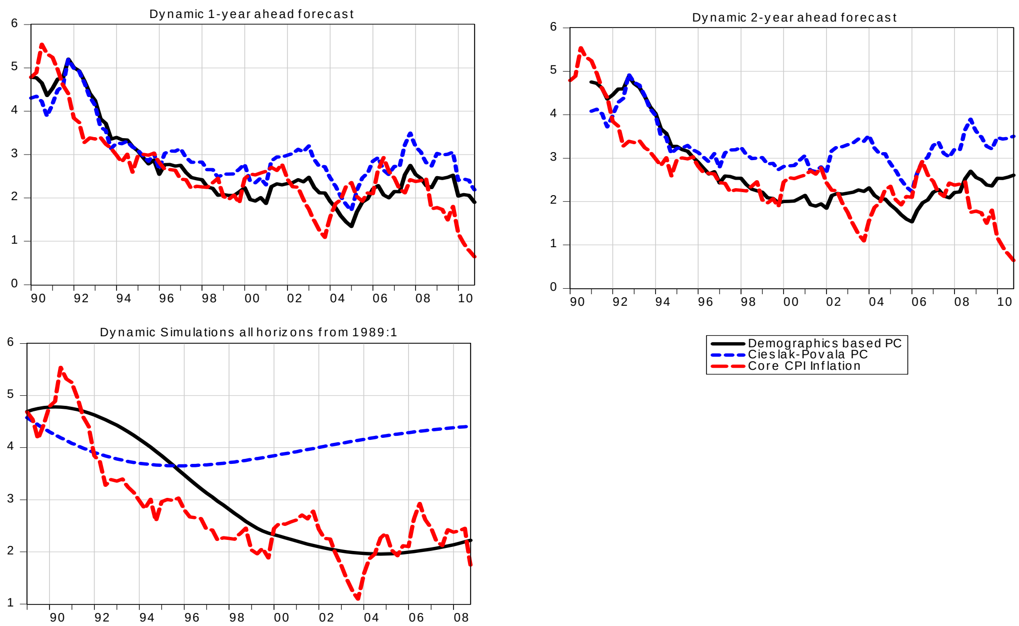

Figure 10. Dynamic out-of sample forecasts at different horizons. We plot actual values for annual Core CPI inflation and out-of sample forecasts based on the model with a permanent component captured by demographics and on the model with the Cieslak-Pavala permanent component. the forecasting horizon is 1-year in the top-left panel , 2-year in the top-right panel, while the bottom panel contains dynamic forecasts in 1990:1 for all horizons starting from 1-quarter ahead.

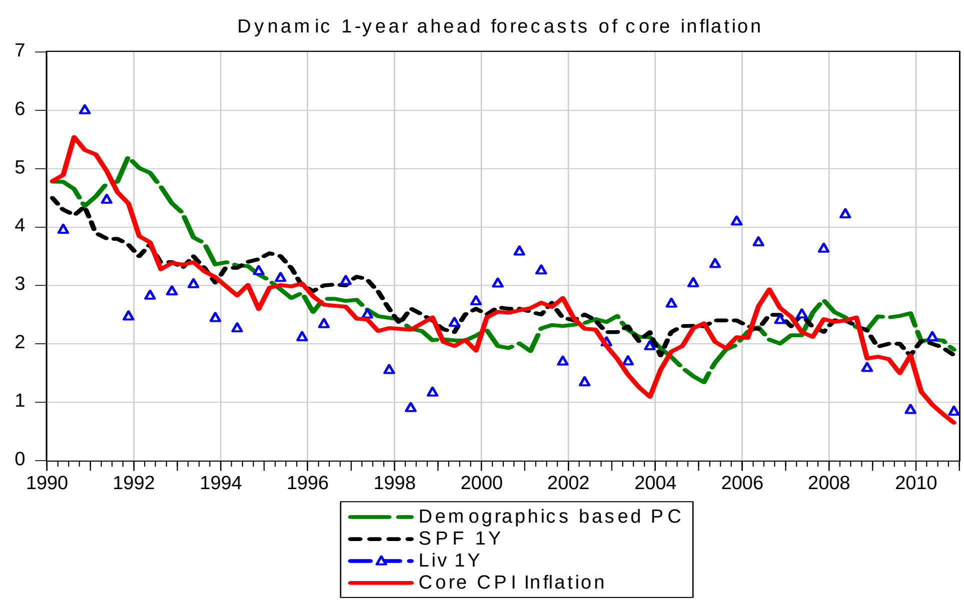

Figure 11. Dynamic 1-year ahead forecasts of core inflation. 1-year ahead forecasts for Core CPI inflation based on the model with demographics and 1-year forecast from the Survey of Professional Forecasters, and the Livingston survey . Median survey response are plotted.

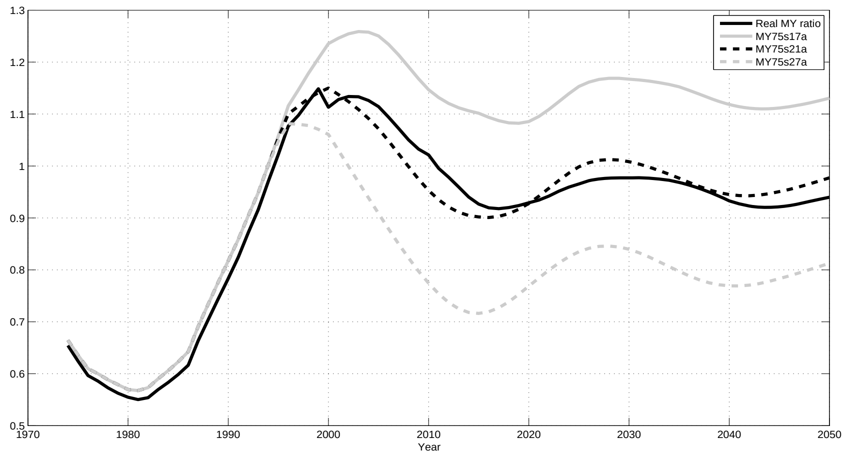

Figure 12. Long term projections of MY ratio. This figure plots actual MY ratio and its long run projections based on alternative scenarios for the fertility rate. The actual MY ratio (solid black line) is based on annual reports of BoC while MY75s17a (solid grey line), MY75s21a (dashed black line) and MY75s27a (dashed grey line) are predicted in 1975 with 1.7, 2.1 and 2.7 fertility rates respectively. All the projection information is from BoC's 1975 population estimation and projections report.

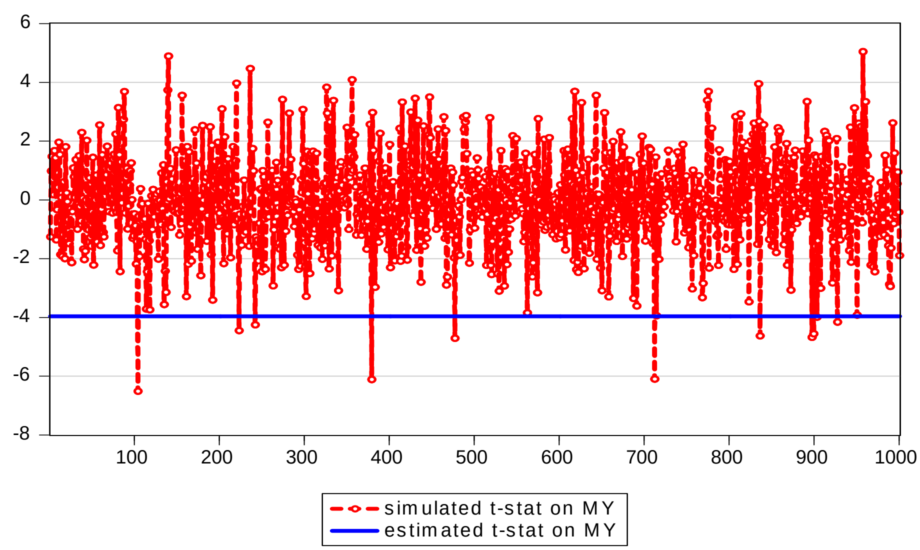

Figure 13. Simulated t-statistics are the t-stat on the MY ratio in an autoregressive model where the dependent variable is an artificial series bootstrapped (1000 simulation) from an autoregressive model for the 3-month rates. The estimated t-stat is the observed value of the t-stat on MY in an autoregressive model for the actual 3-month rate augmented with MY.